Scoring Key
Unit 4: Linear Momentum

Fifty-two rubric points across the five free-response problems of Chapter 4 — and, at each one, the shortest answer that still earns it.

What this key is for

A suggested way to score problems 4.1–4.5 of the AP Physics 1 workbook, to help teachers grade them quickly and consistently. The format follows College Board's published scoring guidelines, but the points and thresholds are this book's own reading of the standard — another reader might draw a line differently.

The problems in this workbook are original, so there is no official rubric for them and this one is the book's own. **It is not a College Board document and is not endorsed by College Board.** AP® is a registered trademark of College Board, which was not involved in producing this material.

How to read one point

Every point carries four things, in this order.

For …	The condition. One sentence saying what the response must contain. The label on the right — Point A1 — names the point; the letter is the part of the problem.
Scoring Note	A ruling on an edge case: what two parts may be scored together, which errors are carried forward, what a bare final answer is worth.
Example Response	A full-credit answer. It is a model, not a threshold — a response can be much thinner than this and still earn the point.
Minimum sufficient	**The threshold.** The least a student can put on the page and still be given the point, plus, where it matters, the near-miss that sits just below it. This is the line published guidelines leave to the reader's judgement; here it is written down so that twenty-five papers can be graded in a quarter of an hour and graded the same way twice.

Three standing rules. A point is earned independently of those around it unless a Scoring Note says otherwise; an error made early and carried through consistently is scored as correct downstream — though leaving out a term the part is about, or working a different collision, such as keeping the kinetic energy of carts that stick, answers a different question and is not carried forward; and a bare answer earns the points its Scoring Note names, never more.

What each problem is worth

The totals are not free choices. They are the point values the 2025 exam gives each question type — ten, twelve, ten and eight, forty on the paper. Chapter 4 has one problem of every type and a second Translation Between Representations problem. The book groups them the way this key does: Set A, problems 4.1–4.4, is a complete ninety-minute free-response section; Set B is a single twelve-point problem that carries the chapter into two dimensions — about twenty-seven minutes, a supplement rather than a second paper.

PROBLEM	TYPE	WHAT IT TESTS	POINTS
Set A — Core			
4.1	MR	Two carts that stick together; the system's momentum through the collision, the shared speed and the kinetic energy lost	**10**
4.2	TBR	A collision with one cart's outcome given; momentum bar charts, velocity graphs, and whether kinetic energy survives	**12**
4.3	LAB	A sticking collision used to find an unknown mass; then a cart's mass read from the slope of impulse against speed	**10**
4.4	QQT	A rubber ball and a clay ball thrown at a wall; why the bounce delivers the larger impulse	**8**
		Set A total	**40**
Set B — Reinforcement			
4.5	TBR	A glancing collision on an air table; momentum kept separately along x and along y	**12**
		Set B total	**12**
		Chapter 4 total	**52**

MR Mathematical Routines • **TBR** Translation Between Representations • **LAB** Experimental Design and Analysis • **QQT** Qualitative Quantitative Translation.

Set A — Core

Problems 4.1–4.4 • 40 rubric points • one of each question type • a full ninety-minute free-response section. The tally sheet for this set is on the last page but one.

Problem 4.1: Mathematical Routines (MR) — 10 rubric points

The problem. Cart A of mass $3m$ moves right at speed v_0 along a frictionless horizontal track toward Cart B of mass m, which is at rest. The carts collide and stick together; the collision lasts from t_1 to t_2. Part (b) repeats the collision with Cart B moving left at $v_0/2$ beforehand.

Part (a) i *Sketch the x-component of the two-cart system's total momentum, p_x, against time, and label its value or values on the vertical axis.*

For a system total that does not move: one flat line above the time axis, at the same height before, during and after the collision. **Point A1**

Scoring Note. What this point tests is that the collision leaves no trace on this graph: the forces between the carts act inside the system, so the total stays put while each cart's own momentum changes. A dip, step or spike anywhere between t_1 and t_2 costs the point, however briefly it is drawn. The line need not start exactly at $t = 0$ or reach the right-hand edge, but it has to be visible on both sides of the collision.

EXAMPLE RESPONSE

A straight horizontal line at height $3mv_0$, from $t = 0$ across the collision interval to the end of the axis, with no change between t_1 and t_2.

MINIMUM SUFFICIENT A single flat line above the time axis that crosses the interval from t_1 to t_2 without a break. **Not enough:** a line that drops during the collision — that is Cart A's momentum, not the system's; **also not enough:** a line drawn along the time axis.

For marking the line's value as $3mv_0$ on the vertical axis. **Point A2**

Scoring Note. Mark A2 without looking at A1: a response that labels $3mv_0$ correctly but draws a dip during the collision keeps A2. A label that is the same value in other symbols — $(3m)v_0$, or $4mv_f$ written at the same height — earns it. Two different values, one before and one after the collision, do not: the whole answer to this part is that there is only one.

EXAMPLE RESPONSE

The line's height is marked $3mv_0$ where it meets the vertical axis.

MINIMUM SUFFICIENT $3mv_0$ written beside the vertical axis at the height of the line. **Not enough:** mv_0 — Cart A's mass is $3m$; **also not enough:** a word such as "constant" in place of a value.

Part (a) ii *Derive the speed v_f of the joined carts just after the collision in terms of m and v_0, beginning from a fundamental principle or a reference equation.*

For opening the derivation with conservation of momentum, written in general symbols before this problem's masses and speeds go in. **Point A3**

Accept any of these openings:

- $p_i = p_f$, or $\sum p_{\text{before}} = \sum p_{\text{after}}$
- $m_A v_{A,i} + m_B v_{B,i} = (m_A + m_B)\, v_f$, or the same with the zero term already dropped, $m_A v_A = (m_A + m_B)\, v_f$
- the impulse–momentum theorem for each cart, with the two impulses the same size and opposite in sense

Scoring Note. The principle has to be written before it is used — the same ruling as every opening point in this series. General symbols with a zero term left out still count. A first line with the carts' own masses in it, $3mv_0 = 4mv_f$, has applied conservation correctly and earns A4, but not this point.

EXAMPLE RESPONSE

The track exerts no horizontal force on the two carts, so the system's momentum is conserved: $p_{x,i} = p_{x,f}$.

MINIMUM SUFFICIENT $p_i = p_f$, in any notation — or the impulse–momentum theorem for each cart, with opposite impulses — written before the masses and speeds are substituted. **Not enough:** "momentum is conserved" in words with no equation; **and not enough:** opening with $\frac{1}{2}(3m)v_0^2 = \frac{1}{2}(4m)v_f^2$ — the carts stick, so this collision is exactly the kind that does not keep its kinetic energy.

For the correct expression $v_f = \frac{3}{4}v_0$. **Point A4**

Accept any equivalent form:

- $\frac{3v_0}{4}$, $0.75\,v_0$, or $\frac{3mv_0}{4m}$ left unsimplified

Scoring Note. A bare correct answer earns A4 alone; A3 needs the principle on the page. The error this point exists to catch is the energy route: setting the two kinetic energies equal gives $v_f = (\sqrt{3}/2)\,v_0 \approx 0.87\,v_0$, a plausible-looking number that is simply wrong for carts that stick.

EXAMPLE RESPONSE

$(3m)v_0 + m(0) = (3m + m)v_f$, so $3mv_0 = 4mv_f$ and $v_f = \frac{3}{4}v_0$.

MINIMUM SUFFICIENT $v_f = 3v_0/4$, or any equivalent, with no working required. **Not enough:** $(\sqrt{3}/2)\,v_0$, from conserving kinetic energy; **also not enough:** $3v_0$, which leaves only Cart B's mass moving after the collision.

Part (a) iii *Derive the magnitude of the kinetic energy lost in the collision, $|\Delta K|$, in terms of m and v_0, beginning from a fundamental principle or a reference equation.*

For opening part (a) iii with the kinetic-energy relation in general symbols and carrying it into the system's two states, before and after the collision. **Point A5**

Accept either opening:

- $K = \frac{1}{2}mv^2$, then written for the system before and after
- $\Delta K = K_f - K_i$, or $K_{\text{lost}} = K_i - K_f$, with $K = \frac{1}{2}mv^2$

Scoring Note. The opening rule of A3 applies: the relation goes down in general symbols before the carts' masses go in. A first line that already carries them, $\frac{1}{2}(4m)v_f^2 - \frac{1}{2}(3m)v_0^2$, is the substitution and counts toward A6 and A7 only. $K = \frac{1}{2}mv^2$ copied from the reference sheet and never used does not earn A5 either — an opening has to open something. A response that writes $K = \frac{1}{2}mv^2$, works out K_i and K_f, and then states the loss without a separate subtraction line has done the whole derivation: A5 is earned, and A6 and A7 on their own terms.

EXAMPLE RESPONSE

$\Delta K = K_f - K_i$, with $K = \frac{1}{2}mv^2$ for the mass that is moving in each state.

MINIMUM SUFFICIENT $K = \frac{1}{2}mv^2$, $\Delta K = K_f - K_i$ or $K_i - K_f$ in general symbols, followed by the kinetic energies of the two states. **Not enough:** a first line with $3m$ and $4m$ already in it; $K = \frac{1}{2}mv^2$ with nothing done to it; $\frac{1}{2}m(\Delta v)^2$, which squares the change in speed instead of each speed — the slip the book warns against in problem 4.2.

For the two kinetic energies with the right mass in each: $K_i = \frac{1}{2}(3m)v_0^2$ before, and $K_f = \frac{1}{2}(4m)v_f^2$ after, with the joined mass $4m$. **Point A6**

Scoring Note. This point is judged on the masses alone, whatever v_f is used: a response that puts its own v_f, right or wrong, into K_f with the mass $4m$ earns A6. Using $3m$ in a K_f that is otherwise there — the right expression with Cart A's mass alone — is the error the book names beside this part, and it costs the point.

EXAMPLE RESPONSE

$K_i = \frac{1}{2}(3m)v_0^2 = \frac{3}{2}mv_0^2$ and $K_f = \frac{1}{2}(4m)\left(\frac{3}{4}v_0\right)^2 = \frac{9}{8}mv_0^2$.

MINIMUM SUFFICIENT Both kinetic energies written with $3m$ before and $4m$ after, whether or not simplified. **Not enough:** K_f with the mass $3m$.

For the correct result $|\Delta K| = \frac{3}{8}mv_0^2$. **Point A7**

Accept either sign convention:

- $|\Delta K| = \frac{3}{8}mv_0^2$, $0.375\,mv_0^2$, or $\dfrac{3mv_0^2}{8}$
- $\Delta K = -\frac{3}{8}mv_0^2$, where the response says or shows that this is energy lost

Scoring Note. A bare correct result carries A6 as well as A7, but not A5. Error carried forward applies to a wrong term that is present: a v_f wrong by a slip, or a wrong mass in a K_f that is there — $3m$, the error A6 has already charged, gives $K_f = \frac{27}{32}mv_0^2$ and a loss of $\frac{21}{32}mv_0^2$, and that response earns A7. As page 1 says, it does not apply to a v_f from a different collision or from a term left out: conserving kinetic energy makes the loss zero by construction, and $v_f = 3v_0$ — Cart A's mass left out after the collision — turns the loss into a gain. Neither earns A7.

EXAMPLE RESPONSE

$\Delta K = \frac{9}{8}mv_0^2 - \frac{12}{8}mv_0^2 = -\frac{3}{8}mv_0^2$, so $|\Delta K| = \frac{3}{8}mv_0^2$.

MINIMUM SUFFICIENT $\frac{3}{8}mv_0^2$, or an equivalent, identified as the energy lost — or the loss the response's own K_i and K_f give, unless its v_f came from conserving kinetic energy or from a term left out, such as $3mv_0 = mv_f$. **Not enough:** $\frac{9}{8}mv_0^2$, the energy left over; **also not enough:** zero, from a v_f found by conserving kinetic energy, or a "loss" worked from $3mv_0 = mv_f$.

Part (b) *Cart B now moves left at $v_0/2$ before the same sticking collision. Indicate whether the joined carts' speed is greater than, less than or equal to v_f, and justify, including qualitative reasoning beyond derivations or expressions.*

For selecting *Less than* v_f. **Point B1**

Scoring Note. The selection alone earns this point, whatever is written beside it.

EXAMPLE RESPONSE

Less than v_f is marked.

MINIMUM SUFFICIENT The *Less than* box marked, ticked or circled, or the choice written out unambiguously in words. **Not enough:** two boxes marked, or no choice at all.

For stating, in words, that the joined mass is the same in both trials, $3m + m = 4m$. **Point B2**

Scoring Note. This is one half of the comparison: after sticking, the shared speed is the system's momentum spread over the joined mass, so the argument has to say which of the two has changed. The mass has to be said, not left inside the algebra — a $4m$ that appears only in $v'_f = p/4m$ does not earn this point. A response that never mentions the mass can still earn B3.

EXAMPLE RESPONSE

Both times the carts end up stuck together, so the mass that moves after the collision is $4m$ in each trial.

MINIMUM SUFFICIENT A statement that the combined mass is unchanged, or $4m$ both times. **Not enough:** a $4m$ that appears only inside an equation; listing the masses without saying they are the same in the two trials.

For explaining that Cart B's leftward momentum cancels part of Cart A's, so the system starts with less total momentum than in part (a) and the joined carts end up slower. **Point B3**

Scoring Note. The stem asks for reasoning beyond derivations, so the calculation $v'_f = \frac{5}{8}v_0 < \frac{3}{4}v_0$ does not earn B2 or B3 on its own, correct as it is. A response that makes the momentum argument in words and then checks it with those numbers earns B3 — and B2 as well, if it says in words that the mass is unchanged: the algebra may support the argument, but it may not be the argument. Comparing speeds instead of momenta is the error the book flags here — Cart B is slower than Cart A, but its momentum points the other way.

EXAMPLE RESPONSE

In part (a) Cart B carried no momentum, so the system had $3mv_0$ to the right. Now Cart B moves left, its momentum points against Cart A's, and the total momentum before the collision is smaller. After sticking, that smaller momentum is shared by the same $4m$, so the joined carts must move more slowly than v_f. (Check: $p = \frac{5}{2}mv_0$, so the speed is $\frac{5}{8}v_0 < \frac{3}{4}v_0$.)

MINIMUM SUFFICIENT One sentence saying the leftward-moving Cart B lowers the system's total momentum, so the joined carts end up slower. **Not enough:** $\frac{5}{8}v_0 < \frac{3}{4}v_0$ with no words; **also not enough:** "more kinetic energy is lost" — true here, but it is not what sets the final speed.

Problem 4.2: Translation Between Representations (TBR) — 12 rubric points

The problem. Cart 1 of mass $2M$ moves right at v_0 along a frictionless track toward Cart 2 of mass M, at rest. After the collision Cart 1 still moves right, at $v_0/3$. Rightward is positive. A momentum bar chart shows the carts before the collision: Cart 1's bar is drawn to the height that stands for $2Mv_0$, and Cart 2's is at zero.

Part (a) *Draw shaded bars for both carts after the collision, starting at the zero line, with heights consistent with the scale set by the before-collision bar.*

For Cart 1's bar: above the zero line and about a third as tall as its before-collision bar. — Point A1

Scoring Note. The chart has no grid, so heights are read against Cart 1's before-collision bar: six units, about 25 mm on the printed page, where a ruler settles a doubtful case. Cart 1's correct bar is two units, about 8 mm. The accepted band stops halfway to the named errors on either side: one unit (about 4 mm) is Cart 1's momentum with its mass taken as M instead of $2M$, and three units (about 13 mm) give the carts equal shares. Cart 1 keeps moving right, so a bar below the zero line — a cart that bounced back — does not earn this point.

EXAMPLE RESPONSE

A shaded bar for Cart 1 rising from the zero line to one third of the height of its before-collision bar, representing $\frac{2}{3}Mv_0$.

MINIMUM SUFFICIENT A positive bar between about a quarter and five-twelfths of the before-collision bar — about 6.3 to 10.5 mm on the printed chart; on a copy printed at another scale, measure the before-collision bar and use the fractions. **Not enough:** a bar of a sixth or less (the mass taken as M), of half or more (equal shares, or no change at all), or one below the zero line.

For Cart 2's bar: above the zero line, about two thirds as tall as Cart 1's before-collision bar, and the taller of the two after-collision bars. — Point A2

Scoring Note. The bars show momentum, not speed. Cart 2 is the faster cart after the collision, at $\frac{4}{3}v_0$, and a response that draws its bar to that speed makes it taller than Cart 1's bar before the collision — the error the book names beside this chart. The correct momentum is $\frac{4}{3}Mv_0$, four units, about 17 mm; the band again stops halfway to the named errors, equal shares at three units and the slip $v_2 = \frac{5}{3}v_0$ at five. Error carried forward: a Cart 2 bar outside the band still earns A2 when it follows from an error already charged — a Cart 1 bar that lost A1, with the pair still adding up to the before-collision bar, or the response's own v_2 from part (b) i where B2 was lost for it — provided it is still positive, taller than Cart 1's after-collision bar, and lower than the before-collision bar. That admits Cart 1 at one unit with Cart 2 at five. It does not admit equal shares (three and three) or no change at all (six and nothing): those earn A3 alone.

EXAMPLE RESPONSE

A shaded bar for Cart 2 rising to two thirds of the height of Cart 1's before-collision bar, representing $\frac{4}{3}Mv_0$.

MINIMUM SUFFICIENT A positive bar between about seven-twelfths and three-quarters of Cart 1's before-collision bar — about 14.7 to 18.9 mm — and taller than Cart 1's after-collision bar; or, carried forward from an error already charged, one that is positive, taller than Cart 1's after-collision bar and lower than the before-collision bar. **Not enough:** a bar at or above the before-collision height, the speed-scaled chart; a bar no taller than Cart 1's.

For conservation shown on the chart: the two after-collision bars together as tall as the before-collision bar, however they share it. — Point A3

Scoring Note. This point is for conservation, and it is earned independently of A1 and A2: two after-collision bars of three units each are wrong in their split but right in their total, and they earn A3 alone — as does six and nothing, the chart of a collision that changed no one's momentum. Add the bars with their signs — one drawn below the zero line counts as negative. The speed-scaled chart described under A2 overshoots the total and does not earn it.

EXAMPLE RESPONSE

The two after-collision bars, of two units and four units, together match the six-unit bar before the collision: the system's momentum is still $2Mv_0$.

MINIMUM SUFFICIENT After-collision bars that together come to about the height of the before-collision bar. **Not enough:** a pair that is visibly taller or shorter in total.

Part (b) i *Starting with conservation of momentum, derive the velocity v_2 of Cart 2 after the collision in terms of M and v_0.*

For opening with conservation of momentum as a general statement. **Point B1**

Scoring Note. The same ruling as 4.1 A3: the principle has to be on the page before the carts' values are put into it.

EXAMPLE RESPONSE

No horizontal force acts on the two-cart system from outside, so $p_{x,i} = p_{x,f}$.

MINIMUM SUFFICIENT $p_i = p_f$ in any notation, written before the substitution. **Not enough:** beginning at $2Mv_0 = \frac{2}{3}Mv_0 + Mv_2$ with nothing above it — the principle applied, not stated; that line counts toward B2 only.

For the correct result $v_2 = \frac{4}{3}v_0$, directed to the right. **Point B2**

Accept any equivalent form:

- $\dfrac{4v_0}{3}$ or $1.33\,v_0$, positive or stated as rightward

Scoring Note. A bare correct answer earns B2 alone. Dropping Cart 1's factor of two after the collision, $2Mv_0 = M(v_0/3) + Mv_2$, gives $\frac{5}{3}v_0$ and is the slip to look for.

EXAMPLE RESPONSE

$(2M)v_0 + M(0) = (2M)\frac{v_0}{3} + Mv_2$, so $Mv_2 = 2Mv_0 - \frac{2}{3}Mv_0 = \frac{4}{3}Mv_0$ and $v_2 = \frac{4}{3}v_0$.

MINIMUM SUFFICIENT $v_2 = \frac{4}{3}v_0$ or an equivalent, with no working required. **Not enough:** $\frac{5}{3}v_0$, or a negative v_2.

Part (b) ii *Determine whether the collision is elastic or inelastic, supporting the answer with a relevant calculation.*

For a calculation comparing the carts' total kinetic energy before and after the collision. **Point B3**

Accept either comparison:

- $K_i = \frac{1}{2}(2M)v_0^2 = Mv_0^2$ against $K_f = \frac{1}{2}(2M)(v_0/3)^2 + \frac{1}{2}M(4v_0/3)^2 = Mv_0^2$
- the speed at which the carts separate after the collision, $v_2 - v_1 = v_0$, set against the speed at which they approached before it, also v_0

Scoring Note. The book compares kinetic energies, and that is the calculation to expect. The second route is correct for a head-on collision and earns the point, but the book does not teach it and no student is expected to know it. Error carried forward from B2, within the limits on page 1: a v_2 wrong by a slip, used correctly in K_f, earns B3; one found with a term left out — Cart 1's momentum after the collision dropped, $2Mv_0 = Mv_2$ — does not carry. Computing $\frac{1}{2}m(\Delta v)^2$ for each cart does not earn B3 either — kinetic energy follows the speed in each state, not the change in speed.

EXAMPLE RESPONSE

$K_i = \frac{1}{2}(2M)v_0^2 = Mv_0^2$, and $K_f = \frac{1}{2}(2M)\left(\frac{v_0}{3}\right)^2 + \frac{1}{2}M\left(\frac{4}{3}v_0\right)^2 = \frac{1}{9}Mv_0^2 + \frac{8}{9}Mv_0^2 = Mv_0^2$.

MINIMUM SUFFICIENT Both totals worked out and set side by side, or the speeds of approach and separation compared — with the response's own v_2, unless it came from a term left out. **Not enough:** a kinetic energy for one cart only; totals worked from a v_2 found with Cart 1's after-collision momentum dropped; $\frac{1}{2}m(\Delta v)^2$ in place of $\frac{1}{2}mv^2$.

For a conclusion that follows from the comparison: *elastic*, because the total kinetic energy is the same before and after. **Point B4**

Scoring Note. Read the conclusion against the response's own numbers: a student whose carried-forward K_f differs from K_i — smaller, or larger, as the slip $v_2 = \frac{5}{3}v_0$ makes it — and who therefore says the collision is not elastic earns B4. A comparison built on a v_2 with a term left out is not carried (page 1), so neither is its conclusion. What never earns it is "elastic because momentum is conserved" — momentum is conserved in this collision whatever its type, and the question is about the one quantity that is not always kept.

EXAMPLE RESPONSE

$K_f = K_i$: no kinetic energy is lost, so the collision is elastic.

MINIMUM SUFFICIENT *Elastic*, tied to the equal kinetic energies or to the equal speeds of approach and separation — or, carried forward, *not elastic* tied to the response's own unequal totals or speeds, unless its v_2 came from a term left out. **Not enough:** *elastic* on its own, or justified by momentum conservation.

Part (c) *Sketch and label the velocities v_1 and v_2 against time; the collision takes a short interval centred on t_c; there is no numerical scale, but the relative values before and after must be right.*

For the v_1 line, labelled: constant and positive before t_c, falling during the short collision interval to a smaller positive constant after it. **Point C1**

Scoring Note. A drop drawn as a vertical step at t_c is accepted — the collision is brief — and so is a steep ramp across a short interval. A change spread over much of the time axis is not: the book warns against new velocities that begin long before or after the collision. Cart 1 keeps moving right, so a line that falls to or below the time axis does not earn C1.

EXAMPLE RESPONSE

A labelled line at v_0 from $t = 0$ to just before t_c, dropping during the brief collision to $v_0/3$, then flat.

MINIMUM SUFFICIENT A labelled line that is flat, drops near t_c, then is flat again at a lower positive level. **Not enough:** a line that ends at or below zero; **also not enough:** an unlabelled line.

For the v_2 line, labelled: zero before t_c, rising during the short interval to a positive constant after it. **Point C2**

Scoring Note. Steps and ramps are judged as in C1. The two lines may cross during the collision — in the book's graph they do, at $\frac{2}{3}v_0$ — but the crossing is not required.

EXAMPLE RESPONSE

A labelled line along the time axis until just before t_c, rising during the collision to $\frac{4}{3}v_0$, then flat.

MINIMUM SUFFICIENT A labelled line on the time axis before the collision and flat at a positive level after it, with the change near t_c. **Not enough:** a v_2 line that starts above zero.

For the level Cart 2 ends at: above Cart 1's starting speed, v_0, and so above both levels of the v_1 line. **Point C3**

Scoring Note. This is the feature the problem is built around: in an elastic collision one cart can leave faster than the other arrived. Cart 1's own drop is C1's business and is not scored again here. Read this point against the response's own part (b) i — a v_2 level consistent with a carried-forward result earns C3. Exact ratios need not be drawn, and labelled values (v_0 and $\frac{4}{3}v_0$) in place of an accurate drawing are accepted.

EXAMPLE RESPONSE

After the collision the v_2 line sits above the level where v_1 started; the levels are marked v_0, $\frac{1}{3}v_0$ and $\frac{4}{3}v_0$.

MINIMUM SUFFICIENT After t_c, the v_2 line visibly higher than the v_1 line was before the collision — or $\frac{4}{3}v_0$ and v_0 labelled. **Not enough:** v_2 ending level with, or below, Cart 1's starting speed when part (b) i gave $\frac{4}{3}v_0$.

Part (d) *A student claims the impulse on Cart 2 must be smaller in magnitude than the impulse on Cart 1, because Cart 2 has the smaller mass. Indicate whether the claim is correct or incorrect, and justify by referring to the momentum bar charts from part (a).*

For selecting *Incorrect*. **Point D1**

Scoring Note. The selection alone earns this point. Unlike a selection that follows from an earlier sketch, this one is not read against the student's bars: the stem itself states that the carts push on each other with forces of equal size at every instant, so the claim is false whatever part (a) shows.

EXAMPLE RESPONSE

Incorrect is marked.

MINIMUM SUFFICIENT The *Incorrect* box marked, ticked or circled, or the choice written out unambiguously in words. **Not enough:** both boxes marked, or no choice at all.

For reading the momentum changes off the bar charts as the impulses: Cart 1's bar falls by as much as Cart 2's rises, so the impulse on Cart 2 is not the smaller — the two are the same size. **Point D2**

Accept the changes read either way:

- from the bars: Cart 1 drops four units, Cart 2 gains four units
- from the values behind them: $\Delta p_1 = -\frac{4}{3}Mv_0$ and $\Delta p_2 = +\frac{4}{3}Mv_0$

Scoring Note. The stem asks for the bar charts, so an argument from the third law alone — equal forces for the same time — is correct physics but does not earn D2: the stem supplied that fact, and restating it is not reading the charts. A response that reads the charts and adds the third law earns it. Error carried forward: read the changes off the student's own bars, with the impulse taken as the change in momentum. A pair that splits wrongly but still sums correctly shows changes of equal size. The speed-scaled chart shows Cart 2 gaining eight units while Cart 1 loses four — wrong, but it still refutes the claim, and a response that reads it that way earns D2. A reading that makes Cart 2's change the smaller supports the claim and does not. The mass decides how much each cart's velocity changes, not the impulse — a good addition, not a substitute.

EXAMPLE RESPONSE

Impulse is the change in momentum. On the charts Cart 1's bar drops from six units to two, a change of four units, and Cart 2's rises from zero to four. The two changes are the same size and point opposite ways, so the impulses have equal magnitudes: the smaller mass gets the larger change in velocity, not the smaller impulse.

MINIMUM SUFFICIENT The two bar changes read as the impulses ($J = \Delta p$ need not be written out), and as equal in size — or, on the response's own chart, Cart 2's read as not the smaller. **Not enough:** Newton's third law restated with no reference to the charts; **also not enough:** "the charts show the claim is wrong", which points at the charts without reading them.

Problem 4.3: Experimental Design and Analysis (LAB) — 10 rubric points

The problem. Students find the mass M of Cart 2 from sticking collisions on a frictionless track: Cart 1, of known mass m_0, is launched at several speeds into Cart 2 at rest, and Velcro joins them. Equipment: the track, the two carts, two photogates with a timer, a metre stick. In a second experiment a cart with a rubber bumper bounces off a wall with its speed unchanged, and a force sensor on the wall records the impulse magnitude $|J|$; the table gives $|J|$ for five approach speeds v from $0.40\,\text{m/s}$ to $2.00\,\text{m/s}$.

Part (a) *Describe a procedure for collecting the data needed to determine M, including steps that reduce experimental uncertainty.*

For a procedure that finds Cart 1's speed before the collision and the joined carts' speed after it, from the photogate timings, at several different launch speeds. **Point A1**

Accept any valid way of turning a gate reading into a speed:

- a length measured with the metre stick — one cart, the joined pair, or a flag — divided by the time it blocks the beam
- any other correct use of the listed equipment that yields both speeds

Scoring Note. Both speeds are needed, and the one after the collision belongs to the two carts moving together: the book's warning beside this part is that the length used at the second gate has to be whatever actually blocks its beam once the carts are joined. A photogate records a time, not a speed; "use the gates to measure the speeds" without saying what length is timed is the common near-miss — the metre stick is on the equipment list for exactly this. Three or more launch speeds is enough, and a procedure that never says how many it uses is read with the stem, which already has Cart 1 launched at various speeds. Each timing has to happen wholly on one side of the collision — Cart 1 clear of the first gate before it touches Cart 2 — but a response that leaves this out keeps the point: the stem does not ask for it.

EXAMPLE RESPONSE

Measure the length of Cart 1, and of the two carts joined, with the metre stick. Put the first photogate far enough before Cart 2 that Cart 1 has passed right through its beam before the carts touch, and the second beyond Cart 2, where only the joined carts reach it. Launch Cart 1 into Cart 2 at rest; record how long Cart 1 blocks the first gate and how long the joined carts block the second, and turn each time into a speed with the matching length. Repeat for at least five different launch speeds.

MINIMUM SUFFICIENT A procedure that measures the speed before and the speed after the collision at several launch speeds (a procedure silent on the number is read with the stem), and says how a gate's time becomes a speed. **Not enough:** only Cart 1's incoming speed; **also not enough:** a procedure built explicitly on one launch speed, which gives one point and no graph.

For a genuine uncertainty-reducing step, stated as an action. **Point A2**

Accept any one of these:

- repeating each launch speed several times and averaging the speeds
- levelling the track
- starting Cart 2 from the same marked position every time
- lining the carts up so they meet head-on and stay on one line

Scoring Note. Repetition alone earns this point, with or without the word "average". The several launch speeds that A1 already requires cannot be counted a second time here. "Be careful", "measure accurately" and "avoid errors" are intentions, not actions, and do not earn it.

EXAMPLE RESPONSE

Repeat each launch speed three times and average the speeds. Keep the track level and start Cart 2 from the same marked spot, so every collision happens at the same place and head-on.

MINIMUM SUFFICIENT One named action that reduces uncertainty — repeating trials counts on its own. **Not enough:** "be accurate", "measure carefully".

Part (b) *Describe how the data could be graphed, and how the graph would be analysed to find M.*

For a pair of quantities whose graph is a straight line, justified from conservation of momentum, $m_0 v_i = (m_0 + M)v_f$. **Point B1**

Accept any pair that the momentum equation makes linear:

- v_i on the vertical axis against v_f on the horizontal, or the reverse
- $m_0 v_i$ against v_f, or the reverse
- $m_0(v_i - v_f)$ against v_f

Scoring Note. The momentum equation is what turns the choice of axes into a derivation rather than a guess, and it may appear here or in part (a). A pair justified by conserving kinetic energy does not earn B1: the carts stick, so kinetic energy is exactly what is not conserved — the points of such a graph may still lie on a line, but its slope would be read wrongly.

EXAMPLE RESPONSE

Momentum is conserved in the collision, $m_0 v_i = (m_0 + M)v_f$, so $v_i = \left(1 + \dfrac{M}{m_0}\right) v_f$. Plot v_i on the vertical axis against v_f on the horizontal axis; the points should fall on a straight line through the origin.

MINIMUM SUFFICIENT The momentum equation, or a sentence saying momentum before equals momentum after, together with a named pair that it makes linear. **Not enough:** a pair justified by conserving kinetic energy.

For showing how M comes out of the slope, with the conversion right for the axes chosen. **Point B2**

Accept the relation that matches the response's axes:

- v_i against v_f: slope $= 1 + M/m_0$, so $M = m_0(\text{slope} - 1)$
- v_f against v_i: slope $= m_0/(m_0 + M)$, so $M = m_0(1/\text{slope} - 1)$
- $m_0 v_i$ against v_f: slope $= m_0 + M$, so $M = \text{slope} - m_0$
- v_f against $m_0 v_i$: slope $= 1/(m_0 + M)$, so $M = 1/\text{slope} - m_0$
- $m_0(v_i - v_f)$ against v_f: the slope is M itself

Scoring Note. Error carried forward from B1 applies to the choice of axes — not to a relation derived from kinetic energy, which is the physics B1 already refused. "The slope gives the mass" without a relation does not earn this point for the book's axes: there the slope is not M at all but $1 + M/m_0$, and the conversion is what the point pays for.

EXAMPLE RESPONSE

Draw a straight line of best fit through the points. Its slope is $1 + M/m_0$, so $M = m_0(\text{slope} - 1)$.

MINIMUM SUFFICIENT A correct written relation between the slope and M for the chosen axes. **Not enough:** "the slope is M" for v_i against v_f, or "use the slope to find M".

Part (c) i *For the wall experiment, indicate two quantities that, when graphed, give a straight line whose slope yields the cart's mass m_c.*

For naming $|J|$ as the vertical-axis quantity and v as the horizontal-axis quantity in the blanks. **Point C1**

Scoring Note. Accept the reverse assignment, and pairs that differ only by a constant factor — $|J|$ against $2v$, for instance — and carry the choice forward through C2 to D2. Where the grid printed under this part comes with its scale already numbered — 0 to 2.50 across and 0 to 3.50 up, which fits only $|J|$ against v — a student who chose another valid pair has to renumber an axis and loses nothing for doing so. $|J|$ against v^2 is not linear here and does not earn C1.

EXAMPLE RESPONSE

Vertical axis: $|J|$, in N · s. Horizontal axis: v, in m/s. For a bounce with unchanged speed the momentum reverses, so $|J| = |\Delta p| = 2m_c v$.

MINIMUM SUFFICIENT $|J|$ and v written in the two blanks. **Not enough:** $|J|$ and v^2.

Part (c) ii *Plot the data, label the axes with names, units and a scale, and draw a straight best-fit line.*

For both axes labelled with the quantity and its unit — $|J|$ in $\mathrm{N \cdot s}$ or $\mathrm{kg \cdot m/s}$, and v in $\mathrm{m/s}$ — and a uniform scale on any axis the student numbered. **Point C2**

Scoring Note. Where the printed grid already carries numbered ticks, the scale half of the instruction is done for the student, and on those axes this point is for names and units. Any axis the student numbered — on a grid without printed numbers, or renumbered for a different pair in (c) i — needs a uniform scale as well.

EXAMPLE RESPONSE

Vertical axis labelled $|J|$ $(\mathrm{N \cdot s})$; horizontal axis labelled v $(\mathrm{m/s})$, each on a uniform scale.

MINIMUM SUFFICIENT Both axes carrying a name and a unit, and a uniform scale on any axis the student numbered. **Not enough:** names without units, or units without names; a scale whose steps change along an axis.

For the five points plotted, at least four of them in the right places for the quantities on the axes. **Point C3**

Scoring Note. On the grid with its printed scale the small squares are 0.1 on each axis; on an axis the student numbered, read them in the student's scale. A point counts as in place when it is within about half a small square of where it belongs; one point out of place still earns C3, two or more lose it.

EXAMPLE RESPONSE

Points at $(0.40, 0.58)$, $(0.80, 1.22)$, $(1.20, 1.77)$, $(1.60, 2.43)$ and $(2.00, 2.97)$.

MINIMUM SUFFICIENT Five points plotted, at least four of them within about half a small square of where they belong for the axes as labelled. **Not enough:** two or more clearly misplaced points.

For one straight best-fit line drawn through the data. **Point C4**

Scoring Note. Straight, and running through the points rather than beside them, is the whole of this point: joining the points dot to dot, the error the book names here, does not earn it. The line need not touch any particular point, and need not pass exactly through the origin.

EXAMPLE RESPONSE

A single straight line through the scatter, close to $|J| = 1.5\,v$, with points on both sides of it.

MINIMUM SUFFICIENT One straight line through the plotted points, with points on both sides of it. **Not enough:** a dot-to-dot chain or a curve.

Part (d) *Using the best-fit line, calculate an experimental value of m_c, showing how the slope was found from two points on the line — not data points.*

For a slope worked out from two points read off the drawn line, with both points identified. **Point D1**

Scoring Note. Two data points in place of two points on the line lose D1 — the stem rules them out by name — but D2 can still be earned on the slope they give, by error carried forward. If the two points chosen happen to lie on the line as well as in the table, award D1.

EXAMPLE RESPONSE

Two points on the line, $(0.20, 0.30)$ and $(2.20, 3.30)$: slope $= \dfrac{3.30 - 0.30}{2.20 - 0.20} = 1.50\,\mathrm{kg}$.

MINIMUM SUFFICIENT Two coordinate pairs stated as read from the line, and the slope computed from them. **Not enough:** a slope with no points shown; **also not enough:** two points that are plainly plotted data.

For an experimental value of m_c, with its unit, obtained from the slope through the relation that matches the axes. **Point D2**

Accept the relation that matches the axes:

- $|J|$ against v: $m_c = \text{slope}/2$
- $|J|$ against $2v$, or $\frac{1}{2}|J|$ against v: $m_c = \text{slope}$
- reversed axes: the relation inverts — for v against $|J|$, $m_c = 1/(2 \times \text{slope})$

Scoring Note. Accept $0.68\,\text{kg}$ to $0.86\,\text{kg}$. The factor of two is the physics of the point: the cart arrives and leaves at the same speed, so its momentum changes by $2m_c v$, and taking the slope itself as the mass — $1.5\,\text{kg}$ on the book's axes — is the error this point exists to catch. The band is wide because the slope is read off a hand-drawn line; a least-squares fit of the table gives $m_c = 0.749\,\text{kg}$. **Error carried forward.** Where D1 was lost for the way the slope was found, D2 is judged on the relation alone: the right relation applied to the response's own slope earns it, inside the band or out of it. Where D1 was earned, the band decides. A value from a single row of the table, $|J|/(2v)$, uses no line and earns nothing here, even though every row lands inside the band.

EXAMPLE RESPONSE

$$m_c = \frac{\text{slope}}{2} = \frac{1.50\,\text{kg}}{2} = 0.75\,\text{kg}.$$

MINIMUM SUFFICIENT A value between $0.68\,\text{kg}$ and $0.86\,\text{kg}$, with the unit, that follows from the response's own slope through the relation for its axes — or, where D1 was lost, whatever that relation gives from the response's own slope. **Not enough:** $1.5\,\text{kg}$, the slope itself on the book's axes; a value from one row of the table, which uses no line; a bare number with no unit.

Problem 4.4: Qualitative Quantitative Translation (QQT) — 8 rubric points

The problem. A rubber ball and a clay ball, each of mass m, are thrown horizontally at the same speed v_0 toward a wall. The rubber ball bounces back; the clay ball sticks to the wall. J_R and J_C are the magnitudes of the impulses the rubber ball and the clay ball deliver to the wall.

Part (a) *Indicate whether J_R is greater than, less than or equal to J_C, and justify in terms of each ball's change in momentum, with qualitative reasoning beyond referencing equations.*

For selecting $J_R > J_C$. — Point A1

Scoring Note. The selection alone earns this point.

EXAMPLE RESPONSE

$J_R > J_C$ is marked.

MINIMUM SUFFICIENT The $J_R > J_C$ box marked, ticked or circled, or the choice written out unambiguously in words. **Not enough:** more than one box marked.

For pointing out what the two balls share before the impact: the same momentum toward the wall — the same mass at the same speed. — Point A2

Scoring Note. Either form earns this point: the same momentum toward the wall, or the same mass and the same speed, which together fix it. Naming only one of mass and speed does not — it leaves room for a difference before the impact.

EXAMPLE RESPONSE

The balls have the same mass and are thrown at the same speed, so each arrives carrying the same momentum toward the wall.

MINIMUM SUFFICIENT The same initial momentum toward the wall, or the same mass and the same speed. **Not enough:** only one of mass and speed; **also not enough:** "they are thrown the same way", which names nothing.

For explaining that the rubber ball's momentum reverses direction — a bigger change than the clay ball's, which is only brought to zero — and that the bigger change in momentum means the bigger impulse. — Point A3

Scoring Note. The question wants reasoning beyond equations, so $2mv_0 > mv_0$ on its own does not earn A2 or A3 — the same ruling as 4.1 (b). The misconception to watch for runs the other way: a response saying the rubber ball's impulse is small, or zero, because it leaves at the speed it came in has treated momentum as a speed rather than a vector, and does not earn A3. How long each ball touches the wall plays no part in this comparison.

EXAMPLE RESPONSE

The clay ball's momentum goes from its incoming value to zero. The rubber ball's momentum has to go to zero and then build up again in the opposite direction, so its change in momentum is larger. The impulse on each ball equals its change in momentum, and each ball gives the wall an impulse of the same size, so the rubber ball delivers the larger impulse.

MINIMUM SUFFICIENT One sentence saying the bounce reverses the momentum — a larger change than stopping — and so gives the larger impulse. **Not enough:** "the rubber ball hits harder", which names no change in momentum.

Part (b) *A ball of mass m arrives at speed v_0 and rebounds at speed v_r, with $0 \le v_r \le v_0$. Starting from the impulse–momentum theorem, derive the magnitude $|J|$ of the impulse it delivers to the wall, in terms of m, v_0 and v_r.*

For opening with the impulse–momentum theorem. **Point B1**

Accept any of these:

- $J = \Delta p$, or $J = p_f - p_i$
- $\vec{J} = \vec{F}_{\text{avg}}\,\Delta t = \Delta\vec{p}$

Scoring Note. As in 4.1 A3, the theorem has to be written before it is used.

EXAMPLE RESPONSE

For the ball, $J = \Delta p = p_f - p_i$.

MINIMUM SUFFICIENT $J = \Delta p$ in any notation, written first. **Not enough:** the theorem named in words only.

For initial and final momenta of opposite sign, with the sign convention stated or plain from the working. **Point B2**

Scoring Note. Either direction may be taken as positive. Giving the rebound momentum the same sign as the incoming one — $p_f = +mv_r$ — is the error the book warns against here; it leads to $m(v_0 - v_r)$, which would make the rubber ball's impulse zero.

EXAMPLE RESPONSE

With the direction toward the wall positive, $p_i = +mv_0$ and, after the bounce, $p_f = -mv_r$.

MINIMUM SUFFICIENT p_i and p_f written with opposite signs. **Not enough:** both positive.

For the correct result $|J| = m(v_0 + v_r)$. **Point B3**

Scoring Note. A bare correct result carries B2 as well, but not B1. Error carried forward from B2 applies: a sign slip in the rebound momentum, carried through correctly, gives $m(v_0 - v_r)$, and that response earns B3 — the slip has been charged once, at B2. Newton's third law takes the magnitude from the ball to the wall; the book says so, but a response need not, to earn the point.

EXAMPLE RESPONSE

$J_{\text{on ball}} = (-mv_r) - (+mv_0) = -m(v_0 + v_r)$. The wall receives an impulse of the same size, so $|J| = m(v_0 + v_r)$.

MINIMUM SUFFICIENT $|J| = m(v_0 + v_r)$, with no working required — or, carried forward from momenta written with the same sign, a slip charged at B2, $m(v_0 - v_r)$. **Not enough:** $m(v_0 - v_r)$ from momenta written with opposite signs, which is a slip at this step; a bare $m(v_0 - v_r)$ with no momenta shown, which earns nothing.

Part (c) *With $v_r = v_0$ for the rubber ball and $v_r = 0$ for the clay ball, indicate whether the expression from part (b) is consistent with the claim in part (a), and briefly justify by referring to the derivation.*

For selecting *Consistent*. **Point C1**

Scoring Note. Award this point on the selection alone. Do not work through the response's algebra to see whether *Not consistent* might be right for it; award *Not consistent* only where the response says in so many words that its own expression disagrees with its part (a) claim — a student who carried $m(v_0 - v_r)$ forward from part (b) and says so honestly is one such case.

EXAMPLE RESPONSE

Consistent is marked.

MINIMUM SUFFICIENT The *Consistent* box marked, ticked or circled, or the choice written out unambiguously in words — or *Not consistent* where the response says outright that its own expression contradicts its part (a) claim. **Not enough:** both boxes marked, or no choice at all.

For a justification that uses the expression: with m and v_0 the same for both balls, $|J|$ grows with v_r, so the bounce gives the larger impulse. **Point C2**

Accept any of these routes:

- the two cases worked out: $J_R = m(v_0 + v_0) = 2mv_0$ and $J_C = m(v_0 + 0) = mv_0$, so $J_R > J_C$
- the dependence stated: $|J|$ increases with v_r, which is larger for the rubber ball
- on a carried-forward $m(v_0 - v_r)$: $|J|$ falls as v_r grows, so $J_R < J_C$ and the expression does not support part (a)

Scoring Note. The justification has to point at the expression, as the stem asks. "It agrees with part (a)" does not, however true it is. A response carrying $m(v_0 - v_r)$ forward from part (b) earns C2 by reading its own expression correctly, whatever that reading says about part (a).

EXAMPLE RESPONSE

In $|J| = m(v_0 + v_r)$ the mass and the approach speed are the same for both balls; only v_r differs. For the rubber ball $v_r = v_0$ and $J_R = 2mv_0$; for the clay ball $v_r = 0$ and $J_C = mv_0$. So $J_R > J_C$, as part (a) claimed.

MINIMUM SUFFICIENT The two impulses worked out from the response's own expression, or a sentence saying how $|J|$ changes with v_r, applied to the two balls. **Not enough:** "yes, consistent" with no reference to the expression.

Set B — Reinforcement

Problem 4.5 • 12 rubric points • a second Translation Between Representations problem, about twenty-seven minutes, that carries momentum conservation into two dimensions — a supplement to Set A, not a paper of its own. Its tally sheet is the last page.

Problem 4.5: Translation Between Representations (TBR) — 12 rubric points

The problem. Puck 1 of mass m moves right at v_0 on a low-friction air table toward an identical Puck 2 at rest. After the collision Puck 1 moves at speed v_1 at 60° above the $+x$-direction and Puck 2 at speed v_2 at 30° below it; $+x$ is to the right and $+y$ up the page, both in the plane of the table. Bar charts show the system's total momentum components before the collision: $p_{x,\text{tot}}$ drawn to the height of mv_0, and $p_{y,\text{tot}}$ at zero.

Part (a) *Draw the after-collision bars for $p_{x,tot}$ and $p_{y,tot}$, starting at the zero line, with any zero component shown as a distinct line on it.*

For the $p_{x,\text{tot}}$ bar after the collision drawn to the same height as the one before, mv_0. — Point A1

Scoring Note. Nothing outside the two pucks pushes along the table, so the total x-momentum cannot change. Judge the height by eye against the before-collision bar. A response that draws each puck's x-momentum as a separate bar, instead of the total the chart asks for, does not earn this point even if the two add up correctly.

EXAMPLE RESPONSE

A shaded $p_{x,\text{tot}}$ bar after the collision exactly as tall as the one before, representing mv_0.

MINIMUM SUFFICIENT An after-collision p_x bar about as tall as the before-collision bar. **Not enough:** a visibly shorter bar — momentum shared out between the pucks is still all there.

For $p_{y,\text{tot}}$ after the collision shown as zero, by a distinct line drawn on the zero line. — Point A2

Scoring Note. The stem asks for a zero to be drawn, not left blank, and that instruction is what tells "zero" apart from "not answered": an empty slot does not earn A2, a line or a written 0 on the zero line does. A bar up or down — usually one puck's y-momentum taken for the system's — is the error the book names beside this chart.

EXAMPLE RESPONSE

A short heavy line along the zero line in the after-collision $p_{y,\text{tot}}$ slot, marked 0.

MINIMUM SUFFICIENT A distinct mark on the zero line — a line, or a written 0. **Not enough:** a blank slot; **also not enough:** a bar above or below the line.

Part (b) i *Starting with conservation of momentum in the y-direction, derive an expression relating v_2 to v_1.*

For opening part (b) with momentum conservation as a statement, at the head of part (b) i or part (b) ii — along y, along x, or as a vector. — Point B1

Scoring Note. This is the principle point for the whole of part (b); B3 below is scored for its content, not for coming first. As in 4.1 A3, the principle has to be written before the components go in. A zero left implicit in a correct y-equation is fine, and parts (b) i and (b) ii are read as one piece of work: the principle written at the start of either, or as the vector statement $\vec{p}_i = \vec{p}_f$, serves both.

EXAMPLE RESPONSE

Momentum along y is conserved, $p_{y,i} = p_{y,f}$, and neither puck moves along y before the collision, so $p_{y,f} = 0$.

MINIMUM SUFFICIENT $p_{y,i} = p_{y,f}$, $p_{x,i} = p_{x,f}$ or the vector $\vec{p}_i = \vec{p}_f$, in any notation, at the head of part (b) i or part (b) ii. **Not enough:** component lines with no statement above them in either part — $mv_1 \sin 60° = mv_2 \sin 30°$ on its own counts toward B2 only.

For the correct relation $v_2 = \sqrt{3}\,v_1$. **Point B2**

Accept any equivalent form:

- $\dfrac{\sin 60^\circ}{\sin 30^\circ}\,v_1$, $\sqrt{3}\,v_1$, or $1.73\,v_1$

Scoring Note. A bare correct relation earns B2 alone. Two slips are common: x-components put into the y-equation, which gives $v_2 = v_1/\sqrt{3}$, and a negative speed carried out of the minus sign — the minus belongs to Puck 2's y-component, not to v_2, as the book is careful to say.

EXAMPLE RESPONSE

$0 = mv_1 \sin 60^\circ - mv_2 \sin 30^\circ$, so $\frac{\sqrt{3}}{2}v_1 = \frac{1}{2}v_2$ and $v_2 = \sqrt{3}\,v_1$.

MINIMUM SUFFICIENT $v_2 = \sqrt{3}\,v_1$ or an equivalent. **Not enough:** $v_2 = v_1/\sqrt{3}$, or $v_2 = -\sqrt{3}\,v_1$.

Part (b) ii *Starting with conservation of momentum in the x-direction and using the result of part (b) i, derive an expression for v_1 in terms of v_0.*

For the x-momentum equation with both final momenta resolved onto the x-axis: $mv_0 = \frac{1}{2}mv_1 + \frac{\sqrt{3}}{2}mv_2$, however the angles are written. **Point B3**

Scoring Note. This point is for the components, and it may be earned by the equation alone. The x-terms are $\frac{1}{2}mv_1$ and $\frac{\sqrt{3}}{2}mv_2$ whichever way the angles are measured — $mv_1 \cos 60^\circ + mv_2 \cos 30^\circ$ from the x-axis, or $mv_1 \sin 30^\circ + mv_2 \sin 60^\circ$ from the y-axis. A response that writes $p_{x,i} = p_{x,f}$ and never resolves the final momenta has not earned B3; one that puts the y-components, $\frac{\sqrt{3}}{2}mv_1$ and $\frac{1}{2}mv_2$, into the x-equation has not earned it either.

EXAMPLE RESPONSE

$p_{x,i} = p_{x,f}$: $mv_0 = mv_1 \cos 60^\circ + mv_2 \cos 30^\circ$.

MINIMUM SUFFICIENT The x-equation with terms equal to $\frac{1}{2}mv_1$ and $\frac{\sqrt{3}}{2}mv_2$. **Not enough:** the y-components in their place, or one puck's term missing.

For the correct result $v_1 = \frac{1}{2}v_0$. **Point B4**

Scoring Note. Error carried forward from B2 and B3 applies to wrong terms that are present. The x-components put into the y-equation in part (b) i give $v_2 = v_1/\sqrt{3}$, and through a correct x-equation $mv_0 = \frac{1}{2}mv_1 + \frac{1}{2}mv_1$, so $v_1 = v_0$; the y-components put into the x-equation give $v_1 = v_0/\sqrt{3}$. Each error was charged once already, and both responses earn B4. It does not apply when a puck's term is left out of the x-equation — that answers a different question, as page 1 says. A bare correct result carries B3 as well.

EXAMPLE RESPONSE

With $v_2 = \sqrt{3}\,v_1$: $mv_0 = \frac{1}{2}mv_1 + m(\sqrt{3}\,v_1)\frac{\sqrt{3}}{2} = \frac{1}{2}mv_1 + \frac{3}{2}mv_1 = 2mv_1$, so $v_1 = \frac{1}{2}v_0$.

MINIMUM SUFFICIENT $v_1 = v_0/2$, or $0.5\,v_0$, with no working required — or the value the response's own relation and x-equation give, with both pucks' terms present. **Not enough:** $v_1 = 2v_0$, from leaving out Puck 2's term; $v_1 = v_0$ from $mv_0 = mv_1$ with nothing resolved, or from assuming Puck 1 keeps its speed; a result that still contains v_2.

Part (b) iii *Determine an expression for v_2 in terms of v_0.*

For the correct result $v_2 = \frac{\sqrt{3}}{2}v_0$. **Point B5**

Accept any equivalent form:

- $\dfrac{\sqrt{3}}{2}\,v_0$, or $0.87\,v_0$

Scoring Note. Error carried forward from B2 and B4 applies to wrong terms that are present — including a negative v_2 that carries B2's sign slip, which has already been charged once. It does not apply to a v_1 found with a puck's term left out, such as $v_1 = 2v_0$: that answers a different question, as page 1 says.

EXAMPLE RESPONSE

$v_2 = \sqrt{3}\left(\frac{1}{2}v_0\right) = \frac{\sqrt{3}}{2}v_0$.

MINIMUM SUFFICIENT $v_2 = (\sqrt{3}/2)\,v_0$ or an equivalent — or the value the response's own B2 and B4 give, with no term left out along the way. **Not enough:** a v_2 that contradicts the response's own relation from part (b) i.

Part (c) *Sketch and label $p_{x,tot}$ and $p_{y,tot}$ against time through a collision centred on t_c, with the right relative values before, during and after it.*

For the $p_{x,\text{tot}}$ line: horizontal at a positive value, unbroken through the collision. **Point C1**

Scoring Note. The same ruling as 4.1 A1: the forces between the pucks are internal, so the total cannot move during the collision, and a dip or step between before and after costs the point.

EXAMPLE RESPONSE

A horizontal line at mv_0 across the whole time axis, with no change near t_c.

MINIMUM SUFFICIENT One flat line above the time axis, continuous through t_c. **Not enough:** any change during the collision.

For the $p_{y,\text{tot}}$ line: on the time axis, at zero, unbroken through the collision, and named. **Point C2**

Scoring Note. A line on the time axis is visible only by its label, so the label is what shows it was drawn: accept $p_{y,\text{tot}}$ written along the axis, or a line traced just above it and named. A bare axis with nothing marked cannot be told from a line never drawn, and does not earn C2. A $p_{y,\text{tot}}$ line that leaves zero during the collision and comes back has the pucks' internal forces changing the system's momentum, and does not earn it either.

EXAMPLE RESPONSE

A line lying along the time axis from start to finish, labelled $p_{y,\text{tot}} = 0$.

MINIMUM SUFFICIENT A line on the time axis through t_c, named as $p_{y,\text{tot}}$. **Not enough:** an unmarked axis; **also not enough:** a line above or below zero at any point.

For the $p_{x,\text{tot}}$ line named and its value marked, mv_0. **Point C3**

Scoring Note. The $p_{y,\text{tot}}$ name is C2's evidence and is not scored again here. This point is for the upper line: its name, and its value — the height its part (a) bar was drawn to.

EXAMPLE RESPONSE

The upper line labelled $p_{x,\text{tot}}$, its height marked mv_0 on the vertical axis.

MINIMUM SUFFICIENT The upper line named $p_{x,\text{tot}}$, with mv_0 marked for it. **Not enough:** an unnamed line, or a value other than mv_0.

Part (d) *A student claims that, because the pucks can move in two dimensions after the collision, both could leave with positive y-components of momentum. Indicate whether the claim is correct, and justify using conservation of momentum in the y-direction and the representations from parts (a) and (c).*

For selecting *Incorrect*. **Point D1**

Scoring Note. The selection alone earns this point; conservation settles it whatever the student drew.

EXAMPLE RESPONSE

Incorrect is marked.

MINIMUM SUFFICIENT The *Incorrect* box marked, ticked or circled, or the choice written out unambiguously in words. **Not enough:** both boxes marked, or no choice at all.

For arguing from the zero total: the system's y-momentum is zero before the collision and must still be zero after it, as the $p_{y,\text{tot}}$ bar and line show, so the two y-components cannot both be positive — one must be negative, or both zero. **Point D2**

Scoring Note. The stem offers two supports, the principle and the representations, and here they say the same thing: $p_{y,\text{tot}}$ is zero after the collision. A justification built on that zero earns D2 whether it points at the bar, at the line or at the conservation law. What does not earn it is "momentum is conserved" with no mention that the total y-component is zero — the claim is false because of the zero, not because of conservation in general. This differs from 4.2 D2, where the third-law route bypasses the charts the stem asks for; here the conservation route is exactly what the charts record.

EXAMPLE RESPONSE

Before the collision neither puck moves along y, so the system's y-momentum is zero, and the bar in part (a) and the flat line in part (c) show it stays zero. If both pucks left with positive y-components, their total would be positive. So at least one puck must leave with a negative y-component of momentum — here Puck 2 does — or both with none.

MINIMUM SUFFICIENT A sentence saying the total y-momentum stays zero, so both components cannot be positive. **Not enough:** "momentum is conserved" alone; **also not enough:** "Puck 2 moves down", which describes this collision without saying why every collision must do the same.

Tally sheet • Set A

Problems 4.1–4.4, forty points — a complete free-response paper. One box per point, in order. Photocopy it, or keep one on the desk and write the student's name at the top. The short labels are reminders, not the rubric — the ruling for each point is on its own page above.

4.1 MR 10 pts

- ☐ **A1** one flat line above zero, unchanged through t_1–t_2
- ☐ **A2** $3mv_0$ marked on the axis
- ☐ **A3** opens with $p_i = p_f$ (or $J = \Delta p$ per cart, impulses opposite)
- ☐ **A4** $v_f = \frac{3}{4}v_0$ — *not* $(\sqrt{3}/2)v_0$
- ☐ **A5** opens with $K = \frac{1}{2}mv^2$ or ΔK, either order, used for both states
- ☐ **A6** K_i with $3m$, K_f with $4m$
- ☐ **A7** $|\Delta K| = \frac{3}{8}mv_0^2$, or ecf — not from a v_f by the energy route or with a term left out
- ☐ **B1** "Less than v_f"
- ☐ **B2** same joined mass, $4m$, said in words
- ☐ **B3** B's leftward p cuts the total — in words

4.2 TBR 12 pts

- ☐ **A1** Cart 1 bar positive, $\frac{1}{4}$–$\frac{5}{12}$ of the before bar (6.3–10.5 mm)
- ☐ **A2** Cart 2 bar $\frac{7}{12}$–$\frac{3}{4}$ of it (14.7–18.9 mm), or ecf if positive, taller than Cart 1's, below the before bar
- ☐ **A3** after bars add up to the before bar
- ☐ **B1** opens with $p_i = p_f$
- ☐ **B2** $v_2 = \frac{4}{3}v_0$
- ☐ **B3** K_i and K_f compared, or approach and separation speeds (ecf, no term left out)
- ☐ **B4** conclusion from its own comparison (no left-out term) — not "momentum is conserved"
- ☐ **C1** v_1: flat, drops at t_c, stays positive
- ☐ **C2** v_2: zero, rises at t_c, flat
- ☐ **C3** v_2 ends above Cart 1's starting speed (or ecf from (b) i)
- ☐ **D1** "Incorrect"
- ☐ **D2** Δp off the bars as impulses: Cart 2's not smaller

4.3 LAB 10 pts

- ☐ **A1** v_i and joined v_f: length ÷ gate time; not one speed only
- ☐ **A2** a real uncertainty-reducing step
- ☐ **B1** linear pair from $m_0v_i = (m_0 + M)v_f$
- ☐ **B2** M from the slope, conversion matching its axes
- ☐ **C1** $|J|$ and v in the blanks
- ☐ **C2** both axes: name and unit (uniform scale where the student numbered)
- ☐ **C3** at least four of the five points in place
- ☐ **C4** one straight line through the points
- ☐ **D1** slope from two points *on the line*
- ☐ **D2** m_c from the slope via its axes; 0.68–0.86 kg if D1 earned

4.4 QQT 8 pts

- ☐ **A1** $J_R > J_C$
- ☐ **A2** same initial p, or same m *and* same v_0
- ☐ **A3** reversal: bigger change in p, so bigger impulse
- ☐ **B1** opens with $J = \Delta p$
- ☐ **B2** p_i and p_f of opposite sign
- ☐ **B3** $|J| = m(v_0 + v_r)$, or ecf from a B2 sign slip
- ☐ **C1** "Consistent" (or a contradiction it states)
- ☐ **C2** reads $|J|$ against v_r in its own expression

SET A TOTAL

4.1 _____ / 10
4.2 _____ / 12
4.3 _____ / 10
4.4 _____ / 8

_____ / **40**

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Tally sheet • Set B

Problem 4.5, twelve points — the chapter's two-dimensional supplement. Same arrangement as Set A: one box per point, in order, with short reminders rather than the rulings themselves.

4.5 TBR 12 pts

- ☐ **A1** p_x after as tall as before, mv_0
- ☐ **A2** p_y after: a drawn zero line, not a blank
- ☐ **B1** momentum conservation stated at the head of (b) i or (b) ii
- ☐ **B2** $v_2 = \sqrt{3}\,v_1$
- ☐ **B3** x-terms $\frac{1}{2}mv_1$ and $\frac{\sqrt{3}}{2}mv_2$
- ☐ **B4** $v_1 = \frac{1}{2}v_0$, or ecf (both pucks' terms present)
- ☐ **B5** $v_2 = \frac{\sqrt{3}}{2}v_0$, or ecf (no term left out)
- ☐ **C1** p_x line flat and positive through t_c
- ☐ **C2** p_y line flat on the time axis, named
- ☐ **C3** p_x line named and marked mv_0
- ☐ **D1** "Incorrect"
- ☐ **D2** total p_y stays zero ⇒ not both positive

SET B TOTAL

4.5 _____ / 12

_____ / **12**

The two sheets together come to fifty-two points, the chapter total. Only Set A is a complete paper; Set B adds one problem for a class that has reached collisions in two dimensions.

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