Scoring Key
Unit 7: Oscillations

Forty-eight rubric points across the five free-response problems of Chapter 7 — and, at each one, the shortest answer that still earns it.

What this key is for

A suggested way to score problems 7.1–7.5 of the AP Physics 1 workbook, to help teachers grade them quickly and consistently. The format follows College Board's published scoring guidelines, but the points and thresholds are this book's own reading of the standard — another reader might draw a line differently.

The problems in this workbook are original, so there is no official rubric for them and this one is the book's own. **It is not a College Board document and is not endorsed by College Board.** AP® is a registered trademark of College Board, which was not involved in producing this material.

How to read one point

Every point carries four things, in this order.

For …	The condition. One sentence saying what the response must contain. The label on the right — Point A1 — names the point; the letter is the part of the problem.
Scoring Note	A ruling on an edge case: what two parts may be scored together, which errors are carried forward, what a bare final answer is worth.
Example Response	A full-credit answer. It is a model, not a threshold — a response can be much thinner than this and still earn the point.
Minimum sufficient	**The threshold.** The least a student can put on the page and still be given the point, plus, where it matters, the near-miss that sits just below it. This is the line published guidelines leave to the reader's judgement; here it is written down so that twenty-five papers can be graded in a quarter of an hour and graded the same way twice.

Three standing rules. A point is earned independently of those around it unless a Scoring Note says otherwise; an error made early and carried through consistently is scored as correct downstream — though leaving out a term the part is about, such as the spring's energy when the cart's speed at half the amplitude is worked out, answers a different question and is not carried forward; and a bare answer earns the points its Scoring Note names, never more.

What each problem is worth

The totals are not free choices. They are the point values the 2025 exam gives each question type — ten, twelve, ten and eight, forty on the paper. Chapter 7 has one problem of every type and a second Qualitative Quantitative Translation problem. The book groups them the way this key does: Set A, problems 7.1–7.4, is a complete ninety-minute free-response section; Set B is problem 7.5, which the book adds as a reinforcement pass on the restoring force behind simple harmonic motion — about eighteen minutes, a supplement rather than a second paper.

PROBLEM	TYPE	WHAT IT TESTS	POINTS
Set A — Core			
7.1	MR	A simple pendulum released from rest; the force diagram at release, the maximum speed, the string's tension at the bottom, and the period on a planet with weaker gravity	**10**
7.2	TBR	A cart on a horizontal spring; energy bar charts, the maximum speed and the speed at $A/2$, K, U_s and E against position, and the speeds at $A/4$ and $3A/4$ compared	**12**
7.3	LAB	A vertical spring's constant from oscillation periods; then k from the slope of T^2 against m	**10**
7.4	QQT	Blocks of mass M and $3M$ on identical springs, compressed equally; which reaches the greater maximum speed	**8**
		Set A total	**40**
Set B — Reinforcement			
7.5	QQT	A block on a spring passing $A/4$ and $3A/4$; where its acceleration is larger, and a as a function of x	**8**
		Set B total	**8**
		Chapter 7 total	**48**

MR Mathematical Routines • **TBR** Translation Between Representations • **LAB** Experimental Design and Analysis • **QQT** Qualitative Quantitative Translation.

Set A — Core

Problems 7.1–7.4 • 40 rubric points • one of each question type • a full ninety-minute free-response section. The tally sheet for this set is on the last page but one.

Problem 7.1: Mathematical Routines (MR) — 10 rubric points

The problem. A simple pendulum, a bob of mass M on a light string of length L, is pulled aside until the string makes a small angle θ_0 with the vertical and is released from rest; the bob then starts a height $h = L(1 - \cos\theta_0)$ above the lowest point of its swing. In the workbook's figure the bob hangs to the right of the pivot. Air resistance is negligible. Part (b) takes the same pendulum to a planet where $g_P = g/4$ and releases it from the same angle.

Part (a) i *On a dot, draw and label arrows for the forces (not components) exerted on the bob at the instant it is released, with relative lengths reflecting relative magnitudes.*

For the two forces on the bob, each drawn from the dot and labelled — the gravitational force straight down, and the string's tension up along the string, toward the pivot — and no others. **Point A1**

Accept any of these labels:

- gravitational force: F_g, Mg, mg, W, weight, force of Earth
- tension: F_T, T, tension, force of string

Scoring Note. The stem asks for forces, not components, so a part of the weight drawn as an arrow of its own — $Mg\sin\theta_0$ along the arc, or $Mg\cos\theta_0$ along the string — counts as an extra force and costs the point; so does a "restoring", "centripetal" or "net" force drawn beside the two real ones. The dot has no string on it, so the tension's direction is read from the figure: the string runs from the bob up to the pivot, and the tension arrow leans from the vertical the same way — up and to the left. The tilt need not match θ_0. A vertical tension arrow is the tension at the lowest point, not at release, and a tension tilted up and to the right puts the bob on the other side of the pivot; neither earns A1. Lengths are A2's business, not this point's.

EXAMPLE RESPONSE

Two labelled arrows from the dot: $F_g = Mg$ straight down, and F_T up and to the left along the line of the string, tilted θ_0 from the vertical.

MINIMUM SUFFICIENT A labelled downward arrow for gravity and a labelled tension arrow pointing up and to the left, toward the pivot, and nothing else. **Not enough:** a vertical tension arrow; **also not enough:** a third arrow — a part of the weight, or a restoring or centripetal force.

For relative lengths: the tension arrow shorter than the gravitational-force arrow. **Point A2**

Scoring Note. At release the bob is at rest, so it has no centripetal acceleration and the forces along the string balance: $F_T = Mg\cos\theta_0$, less than Mg. Tension drawn as long as the weight is the error the book names beside this diagram. The two forces differ by about a fifth at the angle in the figure and by a tenth at the angle of part (a) iii's numbers, so any tension arrow that is visibly the shorter of the two earns A2; by how much is not judged. A2 is judged on the lengths alone, independently of A1, so a response that lost A1 for the tension's direction or for an extra arrow can still earn it. Copies printed before the correction end that box with a sentence saying the tension exceeds the weight *only* at the lowest point; the "only" is wrong, and the line is listed on the workbook's errata page, under page 296. Nothing in this point depends on it.

EXAMPLE RESPONSE

The F_T arrow drawn clearly shorter than the F_g arrow, since $F_T = Mg\cos\theta_0 < Mg$ at release.

MINIMUM SUFFICIENT A tension arrow visibly shorter than the gravity arrow. **Not enough:** two arrows of the same length; **also not enough:** a tension arrow longer than the weight's.

Part (a) ii *Derive the maximum speed $v_{\max}$ of the bob in terms of L, θ_0 and g, beginning from a fundamental principle or a reference equation.*

For opening the derivation with conservation of energy, written in general symbols before the pendulum's own energies go in. **Point A3**

Accept any of these openings:

- $K_i + U_i = K_f + U_f$, or $E_i = E_f$
- the same with the zero terms already dropped, $U_{g,i} = K_f$
- the work–energy theorem written for the bob alone, $\Delta K = \sum W$

Scoring Note. The stem leaves the choice of principle open, and the series' opening rule applies: the principle goes on the page before it is used. A first line that already reads $MgL(1-\cos\theta_0) = \frac{1}{2}Mv_{\max}^2$ has applied conservation correctly and counts toward A4 and A5, but not this point. The tension does no work, since it is always at right angles to the bob's path. Neither constant-acceleration kinematics — the bob's acceleration changes all along the arc — nor $v_{\max} = A\omega$, which the box beside Problem 7.4 calls a derived relation rather than a fundamental principle, earns A3.

EXAMPLE RESPONSE

The tension is perpendicular to the bob's path and does no work, so the mechanical energy of the bob–Earth system is conserved: $K_i + U_{g,i} = K_f + U_{g,f}$.

MINIMUM SUFFICIENT $K_i + U_i = K_f + U_f$, $E_i = E_f$, $U_{g,i} = K_f$ or $\Delta K = \sum W$ — in any notation, before the pendulum's quantities are substituted. **Not enough:** "energy is conserved" in words only; **also not enough:** $v_{\max} = A\omega$.

For the energy equation between release and the lowest point, with the release height in the stem's variables: $MgL(1-\cos\theta_0) = \frac{1}{2}Mv_{\max}^2$. **Point A4**

Scoring Note. The stem gives $h = L(1-\cos\theta_0)$ itself, and this point is for putting it in, on the same line or a later one: $Mgh = \frac{1}{2}Mv_{\max}^2$ with h never written out is one step short. The book sets $U_g = 0$ at the lowest point; any other zero works if it is used at both ends. The slips to look for are the two the book names beside this part — kinetic energy at release, where the bob is at rest, and gravitational energy left at the bottom — and a half dropped from the kinetic energy.

EXAMPLE RESPONSE

Taking $U_g = 0$ at the lowest point: $0 + MgL(1-\cos\theta_0) = \frac{1}{2}Mv_{\max}^2 + 0$.

MINIMUM SUFFICIENT $MgL(1-\cos\theta_0)$ set equal to $\frac{1}{2}Mv_{\max}^2$, in any arrangement — or $Mgh = \frac{1}{2}Mv^2$ with $h = L(1-\cos\theta_0)$ put in anywhere in the work. **Not enough:** $Mgh = \frac{1}{2}Mv^2$ with h left as it is; **also not enough:** a kinetic-energy term at release.

For the correct expression $v_{\max} = \sqrt{2gL(1-\cos\theta_0)}$. **Point A5**

Accept any equivalent form:

- $\sqrt{2gL - 2gL\cos\theta_0}$, or $2\sin(\theta_0/2)\sqrt{gL}$

Scoring Note. A bare correct expression carries A4 as well — there is no way to it without the height in the stem's variables — but not A3. $\sqrt{2gh}$ is the right speed in the wrong variables: the stem asks for L, θ_0 and g, and it does not earn A5. Error carried forward applies to a wrong term that is present in A4's equation: a half dropped from the kinetic energy gives $\sqrt{gL(1-\cos\theta_0)}$, which earns A5. $\theta_0\sqrt{gL}$, from treating the swing as simple harmonic motion with an arc amplitude of $L\theta_0$, is the small-angle approximation of the answer, not an equivalent of it, and on its own does not earn A5; written after the exact form, as a remark on small angles, it costs nothing. The stem does call the angle small, but part (a) iii's numbers put it near 26°, and the exact form is one line away with the height the stem gives.

EXAMPLE RESPONSE

Cancelling M: $gL(1-\cos\theta_0) = \frac{1}{2}v_{\max}^2$, so $v_{\max} = \sqrt{2gL(1-\cos\theta_0)}$.

MINIMUM SUFFICIENT $v_{\max} = \sqrt{2gL(1-\cos\theta_0)}$ or an equivalent, with no working required — or what the response's own energy equation gives. **Not enough:** $\sqrt{2gh}$, in the wrong variables; **also not enough:** $\theta_0\sqrt{gL}$ as the only answer.

Part (a) iii *Calculate the maximum speed of the bob for $L = 1.0\,\text{m}$ and a release height $h = 0.10\,\text{m}$.*

For $v_{\text{max}} = 1.4\,\text{m/s}$, with its unit. **Point A6**

Scoring Note. With h given, $v_{\text{max}} = \sqrt{2gh}$ needs no angle — the book's own route — and $g = 9.8$ or $10\,\text{m/s}^2$ both give $1.4\,\text{m/s}$; finding $\cos\theta_0 = 0.90$ first and using part (a) ii's expression gives the same. Error carried forward from part (a) ii applies: this part checks the substitution, so the value the response's own expression gives earns A6 — $0.99\,\text{m/s}$, for one, from a half dropped from the kinetic energy. $4.4\,\text{m/s}$, or $4.5\,\text{m/s}$ with $g = 10\,\text{m/s}^2$, puts the string's length where the height belongs. Made in this part, that slip is not carried; where part (a) ii already gave $\sqrt{2gL}$, it is that expression's value and earns A6.

EXAMPLE RESPONSE

$v_{\text{max}} = \sqrt{2gh} = \sqrt{2\,(9.8\,\text{m/s}^2)(0.10\,\text{m})} = \sqrt{1.96}\,\text{m/s} \approx 1.4\,\text{m/s}.$

MINIMUM SUFFICIENT $1.4\,\text{m/s}$ — 1.40 or 1.41 — with a unit, or the value the response's own expression gives. **Not enough:** 4.4 or $4.5\,\text{m/s}$ from the length put in for the height in this part; a bare number with no unit.

Part (a) iv *Derive the tension F_T in the string as the bob passes through the lowest point, in terms of M, L, θ_0 and g, beginning from a fundamental principle or a reference equation.*

For opening with Newton's second law for motion on a circle, written in general symbols: the net force toward the pivot equals Mv^2/L. **Point A7**

Accept any of these openings:

- $\sum F = Ma$ with $a_c = v^2/r$, or $\sum F = Mv^2/r$ directly
- $F_{\text{net}} = Ma_c$, or $F_c = Mv^2/r$

Scoring Note. The opening rule of A3 applies. As it passes the bottom the bob is moving on a circle of radius L, so the net force there is not zero: it points up, toward the pivot. Setting the tension equal to the weight, as if the bob hung at rest, is the error the book names beside this part, and the box there names this point by its label: Point A7 is the one missed by students who forget the circular motion. A first line $F_T - Mg = Mv^2/L$ has applied the law correctly and counts toward A8, but not this point.

EXAMPLE RESPONSE

At the lowest point the bob moves on a circle of radius L, so Newton's second law toward the pivot reads $\sum F = \dfrac{Mv_{\text{max}}^2}{L}$.

MINIMUM SUFFICIENT $\sum F = Mv^2/L$, or $\sum F = Ma$ with $a = v^2/L$, in any notation, before the forces are put in. **Not enough:** $\sum F = 0$, or $F_T = Mg$; **also not enough:** $F_T - Mg = Mv^2/L$ with nothing above it, which counts toward A8 only.

For the correct expression $F_T = Mg(3 - 2\cos\theta_0)$, with part (a) ii's speed substituted. **Point A8**

Accept any equivalent form:

- $Mg + 2Mg(1 - \cos\theta_0)$, or $3Mg - 2Mg\cos\theta_0$

Scoring Note. The route is $F_T - Mg = Mv_{\text{max}}^2/L$ with $v_{\text{max}}^2 = 2gL(1 - \cos\theta_0)$ put in; L cancels. A bare correct expression earns A8 alone. Error carried forward applies to the response's own v_{max} from part (a) ii, and to nothing else: from $v_{\text{max}}^2 = gL(1 - \cos\theta_0)$, for one, $Mg(2 - \cos\theta_0)$ earns A8. The force equation is this part's own, and a slip in it costs A8 — $F_T = Mg$, the error named above; the weight left out, $2Mg(1 - \cos\theta_0)$; or a sign slip, $F_T + Mg = Mv^2/L$, which gives $Mg(1 - 2\cos\theta_0)$. An answer that still contains v_{max}, or h, is not yet in the stem's variables. Two points cannot pay for every printed step after the law — the force equation, the substitution and the result — so the result's point carries the other two with it: a correct $F_T = Mg + Mv_{\text{max}}^2/L$, left with v_{max} in it and with no general law above it, scores nothing here. The reason is the budget, not the physics: the first of the two points goes to the law, the point the box beside this part names as Point A7.

EXAMPLE RESPONSE

$F_T - Mg = \dfrac{Mv_{\text{max}}^2}{L}$, and with $v_{\text{max}}^2 = 2gL(1 - \cos\theta_0)$, $F_T = Mg + 2Mg(1 - \cos\theta_0) = Mg(3 - 2\cos\theta_0)$.

MINIMUM SUFFICIENT $F_T = Mg(3 - 2\cos\theta_0)$ or an equivalent, with no working required — or what the response's own v_{max} gives in $F_T = Mg + Mv_{\text{max}}^2/L$. **Not enough:** $F_T = Mg$; an answer that still contains v_{max} or h.

Part (b) *On a planet where $g_P = g/4$, the same pendulum is released from the same angle. Indicate whether its period is greater than, less than or equal to the period on Earth, and justify, including qualitative reasoning beyond referencing equations.*

For selecting *Greater than*. **Point B1**

Scoring Note. The selection alone earns this point, whatever is written beside it.

EXAMPLE RESPONSE

Greater than is marked.

MINIMUM SUFFICIENT The *Greater than* box marked, ticked or circled, or the choice written out unambiguously in words. **Not enough:** two boxes marked, or no choice at all.

For explaining why each swing takes longer: in the weaker field the part of gravity that pulls the bob back toward the bottom is smaller — or, by energy, the same drop gives the bob less speed — so it moves more slowly along the same arc and takes longer over each swing. **Point B2**

Accept either chain:

- through force: a weaker pull back toward the bottom, or a smaller acceleration there, so a slower swing
- through energy: the same drop from the same angle turns less energy into motion, so the bob is slower at every point of the same arc

Scoring Note. The stem asks for reasoning beyond equations, so $T = 2\pi\sqrt{L/g}$ with g in the denominator — or $T_P = 2T_E$ worked out — does not earn B2 on its own, correct as it is: the algebra may support the argument, but it may not be the argument, the same ruling as 5.1 (b) and 6.1 (b). The chain needs both links: why the bob is slower, and that a slower bob takes more time over the same swing. The book argues through the pull; the energy chain is just as sound — a quarter of the field gives half the speed at every angle — and is accepted, as 6.1 B2 accepted a chain that ran through the spin. The stem keeps M, L and θ_0 the same, and the response need not say so. *Equal to*, because the mass has not changed, is the error the book names beside this part: it borrows the period of a mass on a spring.

EXAMPLE RESPONSE

On the planet the gravitational field is a quarter as strong, so the part of gravity that pulls the bob back toward the bottom of its arc, $Mg_P \sin\theta$, is a quarter as large at every point. With the same mass, the bob gains speed more slowly, moves more slowly along the same arc, and takes longer to complete each swing: the period is longer. (Check: $T_P = 2\pi\sqrt{4L/g} = 2T_E$.)

MINIMUM SUFFICIENT One sentence: the weaker field pulls the bob back toward the bottom less strongly — or accelerates it less, or gives it less speed in the same drop — so it swings more slowly and each swing takes longer. **Not enough:** "g is in the denominator, so T is larger"; **also not enough:** "weaker gravity, so a longer period", which names the cause without the mechanism.

Problem 7.2: Translation Between Representations (TBR) — 12 rubric points

The problem. A cart of mass M on a frictionless horizontal track is attached to a spring of constant k, pulled to $x = +A$ and released from rest at $t = 0$; it then oscillates between $x = -A$ and $x = +A$ about equilibrium at $x = 0$. The printed bar chart for $x = +A$ shows K as a line on zero and U_s up to a dashed line marked E; the same dashed E line is printed on the empty charts for $x = 0$ and $x = +A/2$. Part (c) gives axes of energy against position with $-A$, 0 and $+A$ marked and no energy scale.

Part (a) *Draw shaded bars for K and U_s at $x = 0$ and at $x = +A/2$, starting on the zero line, any zero energy drawn as a line on it, heights consistent with the scale.*

For $x = 0$: a K bar reaching the E line, and no spring energy. — **Point A1**

Scoring Note. At equilibrium the spring has its natural length, so $U_s = 0$ and the whole of E is kinetic. On the printed page the dashed E line stands about 25 mm above the zero line, and a K bar within about an eighth of that, some 3 mm, is level with it. A zero left blank rather than drawn as a line on the zero mark costs nothing; a U_s bar of any height costs the point. Each chart is read against its own printed E line, so a mistake at one position does not travel to the other.

EXAMPLE RESPONSE

K a bar up to the E line; U_s a line on zero.

MINIMUM SUFFICIENT A K bar reaching about the E line, and no U_s bar. **Not enough:** a K bar visibly short of the line; **also not enough:** a U_s bar standing at $x = 0$.

For $x = +A/2$: a U_s bar at about a quarter of E, and a K bar making up the rest, so that the two together reach the E line. — **Point A2**

Scoring Note. $U_s = \frac{1}{2}k(A/2)^2$ is a quarter of $\frac{1}{2}kA^2$: the spring's energy goes as the square of the stretch, so half the stretch holds a quarter of the energy. A U_s bar at half of E — the error the book names beside this chart — takes the energy to be proportional to the stretch, and the matching K bar at half carries the same error. The correct bars stand about 6 and 19 mm tall. Accept U_s from about an eighth to about three-eighths of E, 3 to 9 mm: the upper end lies halfway to the half, and the lower end the same distance below a quarter. The pair has to reach the E line to within about an eighth of its height, some 3 mm, short or over.

EXAMPLE RESPONSE

U_s at a quarter of E, about 6 mm; K at three-quarters, about 19 mm; together they reach the E line.

MINIMUM SUFFICIENT U_s between about an eighth and three-eighths of the way up to E, and K filling the rest, so that the pair reaches about the E line. **Not enough:** U_s and K each at half of E; **also not enough:** a pair that falls visibly short of the E line, or overshoots it.

Part (b) i *Starting with conservation of energy, derive the maximum speed v_{max} of the cart in terms of M, k and A, beginning from a fundamental principle or a reference equation.*

For opening with conservation of energy, written as a statement before it is applied. — **Point B1**

Scoring Note. The stem names the principle, so the series' ruling for every "starting with conservation of energy" applies (3.2 B1): accept $E_i = E_f$, the energies at $x = +A$ set equal to those at $x = 0$, the same with the zero terms already dropped ($U_{s,i} = K_f$), or the work–energy theorem written for the cart alone, $\Delta K = \sum W$, with the spring's work in it. A first line that already reads $\frac{1}{2}kA^2 = \frac{1}{2}Mv_{max}^2$ counts toward B2 but not this point. $v_{max} = A\omega$ with $\omega = \sqrt{k/M}$ reaches the right answer from a derived relation — the box beside Problem 7.4 says so — and earns what a bare answer earns, B2 and B3, never B1. Read (b) i and (b) ii as one piece of work: the principle, written once at the head of either, serves both.

EXAMPLE RESPONSE

Conservation of energy for the cart–spring system: $K_i + U_{s,i} = K_f + U_{s,f}$.

MINIMUM SUFFICIENT $E_i = E_f$, the energies at the two positions set equal, or $\Delta K = \sum W$ — in any notation. **Not enough:** "energy is conserved" in words only.

For the energies at the two ends: all spring energy at $x = +A$, all kinetic at $x = 0$ — $\frac{1}{2}kA^2 = \frac{1}{2}Mv_{\text{max}}^2$. **Point B2**

Scoring Note. At $x = +A$ the cart is at rest, and at $x = 0$ the spring has its natural length, so one term survives on each side. The half belongs on both sides or on neither; a half dropped from one side is the slip to look for, and it costs this point.

EXAMPLE RESPONSE

$0 + \frac{1}{2}kA^2 = \frac{1}{2}Mv_{\text{max}}^2 + 0.$

MINIMUM SUFFICIENT $\frac{1}{2}kA^2 = \frac{1}{2}Mv_{\text{max}}^2$, in any arrangement. **Not enough:** a half on one side only, or kA for the spring's energy; **also not enough:** a spring term left at $x = 0$.

For the correct expression $v_{\text{max}} = A\sqrt{\frac{k}{M}}$. **Point B3**

Accept any equivalent form:

- $\sqrt{kA^2/M}$, or $\sqrt{k/M}\,A$

Scoring Note. A bare correct expression carries B2 as well, but not B1. Error carried forward applies to a wrong term that is present in B2's equation: a half dropped from the spring side gives $A\sqrt{2k/M}$, and from the kinetic side $A\sqrt{k/(2M)}$; each earns B3.

EXAMPLE RESPONSE

$kA^2 = Mv_{\text{max}}^2$, so $v_{\text{max}} = A\sqrt{\frac{k}{M}}$.

MINIMUM SUFFICIENT $v_{\text{max}} = A\sqrt{k/M}$ or an equivalent, with no working required — or what the response's own B2 equation gives. **Not enough:** $v_{\text{max}}^2 = kA^2/M$, since the question asks for v_{max}.

Part (b) ii *Derive the speed of the cart at $x = A/2$, in terms of M, k and A.*

For an energy equation between $x = +A$ and $x = A/2$, with the spring's energy at $A/2$ squared correctly: $\frac{1}{2}kA^2 = \frac{1}{2}Mv^2 + \frac{1}{2}k(A/2)^2$. **Point B4**

Accept any of these:

- the equation above, with $\frac{1}{2}k(A/2)^2$ written as $\frac{1}{8}kA^2$, or as $E/4$
- the kinetic energy read off part (a)'s chart, $\frac{1}{2}Mv^2 = \frac{3}{4}\cdot\frac{1}{2}kA^2$
- an equation from $x = 0$, $\frac{1}{2}Mv_{\text{max}}^2 = \frac{1}{2}Mv^2 + \frac{1}{2}k(A/2)^2$, with part (b) i's v_{max}

Scoring Note. This part's stem does not ask for a principle to be written first; (b) i's opening serves. $(A/2)^2$ is $A^2/4$, not $A/2$ — the slip the book names beside this part — and a spring energy at $A/2$ taken as half of E is part (a)'s error again; either costs B4. A response that leaves the spring term out altogether, $\frac{1}{2}kA^2 = \frac{1}{2}Mv^2$, has found v_{max} a second time and does not earn it.

EXAMPLE RESPONSE

$\frac{1}{2}kA^2 = \frac{1}{2}Mv^2 + \frac{1}{2}k\left(\frac{A}{2}\right)^2 = \frac{1}{2}Mv^2 + \frac{1}{8}kA^2.$

MINIMUM SUFFICIENT $\frac{1}{2}kA^2$ set equal to $\frac{1}{2}Mv^2$ plus $\frac{1}{8}kA^2$, in any arrangement — or $\frac{1}{2}Mv^2 = \frac{3}{8}kA^2$ directly. **Not enough:** the spring term written as $\frac{1}{2}k\frac{A}{2}$, or as $\frac{1}{4}kA^2$; **also not enough:** no spring term at $A/2$.

For the correct expression $v = \frac{A}{2}\sqrt{\frac{3k}{M}}$. **Point B5**

Accept any equivalent form in M, k and A:

- $A\sqrt{\frac{3k}{4M}}$, $\frac{\sqrt{3}}{2}\,A\sqrt{\frac{k}{M}}$, or $0.87\,A\sqrt{k/M}$

Scoring Note. A bare correct expression carries B4 as well. The stem asks for M, k and A, so $\frac{\sqrt{3}}{2}v_{\max}$ with $v_{\max}$ left as a symbol is one substitution short and does not earn B5 — the ruling of 3.2 B4. Error carried forward applies to a wrong term that is present in B4's equation: a spring energy of half of E gives $A\sqrt{k/(2M)}$, about $0.71\,v_{\max}$, which earns B5. As page 1 says, it does not reach $v = A\sqrt{k/M}$ from an equation with the spring term left out. $v_{\max}/2$ is the guess that the speed falls in step with x; it earns nothing as a guess, but the same number earns B5 where it comes out of the response's own B4 equation — a part (a) chart with the two bars swapped, $K = E/4$ at $A/2$, read into B4, gives it.

EXAMPLE RESPONSE

$\frac{1}{2}Mv^2 = \frac{3}{8}kA^2$, so $v^2 = \frac{3kA^2}{4M}$ and $v = \frac{A}{2}\sqrt{\frac{3k}{M}}$, about $0.87\,v_{\max}$.

MINIMUM SUFFICIENT $v = (A/2)\sqrt{3k/M}$ or an equivalent in M, k and A, with no working required — or what the response's own B4 equation gives, unless the spring's energy at $A/2$ is left out of it. **Not enough:** $\frac{\sqrt{3}}{2}v_{\max}$ with $v_{\max}$ unsubstituted; **also not enough:** $v_{\max}/2$ from taking the speed to fall in step with x, with no energy equation behind it.

Part (c) *On axes of energy against position x, sketch and label curves for K, U_s and the total mechanical energy E of the cart–spring system, from $x = -A$ to $x = +A$.*

For a labelled level line for E, above the axis, running from $x = -A$ to $x = +A$. **Point C1**

Scoring Note. The track is frictionless, so the total does not change anywhere along the motion. The axes carry no energy scale, so the line's height is the student's choice — and it becomes the height the two curves are judged against in C2 and C3. Each of the three lines in this part is scored on its own.

EXAMPLE RESPONSE

A horizontal line labelled E across the whole range, from $-A$ to $+A$.

MINIMUM SUFFICIENT A labelled level line above the axis, spanning about $-A$ to $+A$. **Not enough:** a line that slopes or curves; an unlabelled line.

For a labelled U_s curve shaped like an upright bowl: zero at $x = 0$, rising more and more steeply on both sides to meet the E line at $x = -A$ and $x = +A$. **Point C2**

Scoring Note. $U_s = \frac{1}{2}kx^2$ is a parabola opening upward; its exact shape is not judged, but it has to bend upward, growing steeper toward $\pm A$. A V of two straight lines is part (a)'s error in another form — the spring's energy taken as proportional to the stretch — and does not earn C2. Read the ends against the response's own E line: the curve has to reach it at $\pm A$, not stop below it or run past it. Where no E line was drawn, the ends of the U_s curve and the top of the K curve should stand at about the same height.

EXAMPLE RESPONSE

A parabola labelled U_s, with its lowest point, zero, at $x = 0$, rising symmetrically to touch the E line at $-A$ and at $+A$.

MINIMUM SUFFICIENT A labelled curve, bending upward on both sides, from zero at $x = 0$ to the E line at $\pm A$. **Not enough:** a V of straight lines; **also not enough:** a curve that misses the E line at $\pm A$, or misses zero at $x = 0$.

For a labelled K curve shaped like an upturned bowl: highest, at the E line, at $x = 0$, and falling to zero at $x = -A$ and $x = +A$. **Point C3**

Scoring Note. $K = E - U_s$, so the K curve is the U_s curve turned upside down under the E line, the two always adding up to E. An inverted V of straight lines does not earn C3, for the reason given under C2. C2 and C3 are judged independently, each against the response's own E line.

EXAMPLE RESPONSE

A downward-opening parabola labelled K, touching the E line at $x = 0$ and reaching zero at $-A$ and $+A$ — the mirror image of the U_s curve.

MINIMUM SUFFICIENT A labelled curve, bending downward on both sides, from the E line at $x = 0$ to zero at $\pm A$. **Not enough:** an inverted V of straight lines; **also not enough:** a K curve that stays below E at $x = 0$, or above zero at $\pm A$.

Part (d) *Indicate whether the speed at $x = A/4$ is greater than, less than or equal to the speed at $x = 3A/4$, and justify by referring to the energy curves sketched in part (c).*

For a selection that follows from the response's own part (c) curves — *Greater than* where those curves are right. **Point D1**

Scoring Note. Read this against the curves the student drew, not against the right ones, as in 3.2 D1 and 6.2 D1. Wherever the K curve falls from the middle toward $\pm A$ — even as straight lines — it is higher at $A/4$ than at $3A/4$, and *Greater than* follows. A response whose K curve rises away from the middle, with the labels swapped for instance, makes *Less than* the consistent choice: award D1, the curves having already cost C2 and C3. Where part (c) is blank, award D1 for *Greater than*. A K curve drawn level at the height of E leaves the spring's share out; on its own terms the two speeds are equal, and *Equal to* earns D1 — an exception to page 1's rule on a term left out, made because part (d) asks only that the answer agree with the student's own curves, as in 6.2 D1; D2 follows it. *Equal to*, because the two points are the same distance from the middle of the range, is the error the book names beside this part: it mixes up $A/4$ and $3A/4$ with $-A/4$ and $+A/4$, does not follow from any curve that peaks at $x = 0$, and does not earn D1.

EXAMPLE RESPONSE

Greater than is marked.

MINIMUM SUFFICIENT One box marked, consistent with the response's own K curve — *Greater than* on a correct one. **Not enough:** two boxes marked, or none.

For reading the answer off the curves: U_s is lower at $A/4$ than at $3A/4$, so $K = E - U_s$ is larger there — the K curve is higher at $A/4$ — and more kinetic energy for the same cart means more speed. **Point D2**

Scoring Note. The stem asks for the curves, so $v = \sqrt{k(A^2 - x^2)/M}$ worked out at the two points does not earn D2 on its own, however correct; it may be added as a check — the same ruling as 3.2 D2 and 6.2 D2. Reading the energies the curves stand for counts as reading the curves: "U_s is $E/16$ at $A/4$ and $9E/16$ at $3A/4$, so K is $15E/16$ against $7E/16$" is enough without the word "graph". The cart is the same at both points, so the step from kinetic energy to speed may stay unspoken. Error carried forward: read the argument against the response's own curves — a student whose K curve is higher at $3A/4$, labels swapped, and who argues from it that the cart is faster there, earns D2, and so does one who reads a level K curve as giving equal speeds (D1's note). Copies printed before the correction gloss the speed formula, in the box closing part (b) ii, as depending on how far the cart is from the amplitude rather than from equilibrium; the gloss is wrong, and it is listed on the workbook's errata page, under page 301. An argument from the distance to the turning point reads nothing off the curves and does not earn D2.

EXAMPLE RESPONSE

On my curves E is level, and U_s at $x = A/4$ is far below its value at $3A/4$ — the bowl is shallow near the middle and steep near the ends. So $K = E - U_s$ is larger at $A/4$: the K curve is higher there than at $3A/4$. Same cart, more kinetic energy, so the speed at $A/4$ is greater. (Check: $15E/16$ against $7E/16$.)

MINIMUM SUFFICIENT One sentence: the K curve is higher at $A/4$ than at $3A/4$ — or U_s is lower there, so $K = E - U_s$ is larger — so the cart is faster at $A/4$; or, read against the response's own curves, the speeds they give — faster at $3A/4$ if its K curve is higher there, equal if that curve is level. **Not enough:** the speed formula worked out at both points with no words; **also not enough:** "$A/4$ is closer to equilibrium, so it is faster", which reads nothing off the curves.

Problem 7.3: Experimental Design and Analysis (LAB) — 10 rubric points

The problem. Students determine the spring constant k of a vertical spring. Equipment: the spring on a rigid support, a set of blocks of known masses, a stopwatch and a metre stick. A block of mass m hung from the spring, pulled down slightly and released, oscillates vertically with period T. In a separate experiment, a student times 10 oscillations for each of five masses from $0.100\,\text{kg}$ to $0.500\,\text{kg}$ and divides by 10; the table gives T and leaves a third column to complete if needed.

Part (a) *Describe a procedure for collecting the data needed to determine k, including steps that reduce experimental uncertainty.*

For a procedure that measures the period of oscillation for each of several known masses hung from the spring — or, as a static design, the spring's stretch under each of several known masses. **Point A1**

Scoring Note. $T = 2\pi\sqrt{m/k}$, so the period changes with the hanging mass, and part (b) asks for a graph: the procedure has to give more than one pair of readings. One mass, however often timed, fixes k in principle but leaves part (b) nothing to graph — the same ruling as 4.3 A1, 5.3 A1 and 6.3 A1 — and does not earn A1. The period comes from the stopwatch; how it is timed is A2's business. The stem shows an oscillating block, but it asks only for data that determine k, and the metre stick is on the list: hanging several known masses and measuring how far each stretches the spring is a sound design too, since a hanging block at rest has $kx = mg$, and it earns A1 just as well.

EXAMPLE RESPONSE

Hang a block of known mass from the spring and let it come to rest. Pull it down a few centimetres and release it. Time 10 complete oscillations with the stopwatch and divide by 10 to get the period. Repeat for at least five different masses from the set.

MINIMUM SUFFICIENT The period measured for several different known masses — or the stretch measured for several masses. **Not enough:** one mass only, however often timed; **also not enough:** a procedure that never says what is measured.

For a genuine uncertainty-reducing step, stated as an action. **Point A2**

Accept any one of these:

- timing several oscillations together — the book times ten — and dividing by their number
- repeating each timing and averaging the results
- starting and stopping the stopwatch as the block passes a mark at its equilibrium position, where it moves fastest
- spreading the masses over as wide a range as the set allows, said in so many words
- for the static design, reading the metre stick at eye level, or repeating each reading and averaging

Scoring Note. The first is the book's own step, and the box beside this part names the error it prevents: a single oscillation timed on its own, where the stopwatch's reaction time is a large part of the reading. Repetition alone earns this point, with or without the word "average". The several masses that A1 already requires cannot be counted a second time — the same ruling as 4.3 A2, 5.3 A2 and 6.3 A2 — but a stated spread can, even in the same sentence. A small amplitude is already in the stem and does not earn it; "be careful" and "measure accurately" are intentions, not actions.

EXAMPLE RESPONSE

For each mass, time 10 complete oscillations and divide by 10; do it three times and average. Start and stop the stopwatch as the block passes a marker at its equilibrium position.

MINIMUM SUFFICIENT One named action that reduces uncertainty — timing several oscillations and dividing counts on its own. **Not enough:** "measure carefully"; **also not enough:** "use several masses", or a list of them, which A1 has already paid for.

Part (b) *Describe how the data could be graphed, and how the graph would be analysed to find k.*

For a pair of quantities that plot as a straight line with k in the slope — for the period design, a pair the period relation makes linear. **Point B1**

Accept any such pair:

- T^2 against m, either way round, or the same with a constant factor, such as $T^2/4\pi^2$ against m
- T against $\sqrt{m}$, either way round
- for the static design, the hanging weight mg, or the mass, against the stretch, either way round

Scoring Note. Squaring $T = 2\pi\sqrt{m/k}$ gives $T^2 = (4\pi^2/k)\,m$: T^2 is proportional to m, with k in the slope — the step the box beside this part teaches. Naming a pair it makes linear is enough for B1, as in every LAB of this series. T against m curves and does not earn it.

EXAMPLE RESPONSE

Plot T^2 on the vertical axis against m on the horizontal axis. Since $T^2 = \dfrac{4\pi^2}{k}\,m$, the points fall on a straight line through the origin whose slope contains k.

MINIMUM SUFFICIENT T^2 and m named as the two axes, either way round — or another pair that plots straight with k in the slope; the list gives the usual ones. **Not enough:** T against m.

For showing how k comes out of the slope, with the conversion right for the axes chosen. **Point B2**

Accept the relation that matches the response's axes:

- T^2 against m: slope $= 4\pi^2/k$, so $k = 4\pi^2/\text{slope}$
- m against T^2: slope $= k/4\pi^2$, so $k = 4\pi^2 \cdot \text{slope}$
- T against $\sqrt{m}$: slope $= 2\pi/\sqrt{k}$, so $k = 4\pi^2/\text{slope}^2$; reversed, $k = 4\pi^2 \cdot \text{slope}^2$
- static: mg against the stretch, $k =$ slope; the stretch against mg, $k = 1/\text{slope}$; the stretch against m, $k = g/\text{slope}$; m against the stretch, $k = g \cdot \text{slope}$

Scoring Note. Error carried forward from B1 applies to the choice of axes. "The slope gives k" without a relation does not earn this point: on the book's axes the slope is $4\pi^2/k$, not k, and the conversion is what the point pays for.

EXAMPLE RESPONSE

Draw a straight line of best fit. Its slope is $\dfrac{4\pi^2}{k}$, so $k = \dfrac{4\pi^2}{\text{slope}}$.

MINIMUM SUFFICIENT A correct written relation between the slope and k for the chosen axes. **Not enough:** "the slope is k", or "use the slope to find k".

Part (c) i *For the separate experiment's data, indicate two quantities that, when graphed, give a straight line whose slope yields k, and complete the third column of the table if needed.*

For naming T^2 as the vertical-axis quantity and m as the horizontal-axis quantity in the blanks. **Point C1**

Scoring Note. Accept the reverse assignment, pairs that differ only by a constant factor, and T against $\sqrt{m}$ — the third column then holds $\sqrt{m}$ — and carry the choice forward through C2 to D2, as in 4.3 C1. The static pairs of part (b) do not fit these data, which hold periods. The third column is not scored on its own: C3 checks it through the plotted points, the series' ruling for every table column so far. This part asks part (b)'s question again, and each answer is marked on its own: C1 is not read against B1.

EXAMPLE RESPONSE

Vertical axis: T^2, in s^2. Horizontal axis: m, in kg. The third column reads 0.20, 0.40, 0.59, 0.79 and 1.00.

MINIMUM SUFFICIENT T^2 and m in the two blanks, either way round — or another pair the note accepts. **Not enough:** T and m; **also not enough:** T^2 and $1/m$.

Part (c) ii *Plot the data, label the axes with names, units and a scale, and draw a straight best-fit line.*

For both axes labelled with the quantity and its unit — for the book's pair, T^2 in s^2 and m in kg — and a uniform scale on any axis the student renumbered. **Point C2**

Scoring Note. The grid printed under this part comes numbered, 0 to 0.60 across and 0 to 1.2 up, and those numbers fit the book's pair the book's way round, so on it the scale is done for the student and this point is for names and units. A response that turns the axes round, or plots T against $\sqrt{m}$ — which runs past 0.60 — has to renumber, and a renumbered axis needs a uniform scale. s as the unit of T^2 is the slip to look for.

EXAMPLE RESPONSE

Vertical axis labelled T^2 (s^2); horizontal axis labelled m (kg).

MINIMUM SUFFICIENT Both axes carrying a name and a unit, and a uniform scale on any axis the student renumbered. **Not enough:** names without units, or units without names; s as the unit of T^2.

For the five points plotted, at least four of them in the right places for the quantities on the axes. **Point C3**

Scoring Note. On the printed grid each small square is 0.05 kg across and $0.1\,\mathrm{s}^2$ up, and every mass falls on a grid line, so a point counts as in place within about half a small square of its T^2. The points come from the third column, and this is where the column is checked: a wrong square, plotted faithfully, misplaces its point. The commonest slip, T put where T^2 belongs, misplaces four of the five — all but the last, where $T = 1.00$ squares to itself. One point out of place still earns C3; two or more lose it.

EXAMPLE RESPONSE

Points at $(0.100,\ 0.20)$, $(0.200,\ 0.40)$, $(0.300,\ 0.59)$, $(0.400,\ 0.79)$ and $(0.500,\ 1.00)$, with m across and T^2 up.

MINIMUM SUFFICIENT Five points plotted, at least four of them within about half a small square of where they belong for the axes as labelled. **Not enough:** two or more clearly misplaced points — most often T plotted in place of T^2.

For one straight best-fit line drawn through the data. **Point C4**

Scoring Note. Straight, and running through the points rather than beside them, is the whole of this point: joining the points dot to dot, the error the book names beside this graph, does not earn it. These points lie very close to a line, so a good line may touch all five. The relation puts it through the origin, and the book's own line passes through it, but the student's line need not.

EXAMPLE RESPONSE

A single straight line through the points, close to $T^2 = 2.0\,m$, passing near the origin.

MINIMUM SUFFICIENT One straight line running through the plotted points, not beside them. **Not enough:** a dot-to-dot chain or a curve.

Part (d) *Using the best-fit line, calculate an experimental value of k, showing how the slope was found from two points on the line — not data points.*

For a slope worked out from two points on the drawn line, with both points identified. **Point D1**

Scoring Note. Two data points in place of two points on the line lose D1 — the stem rules them out by name — but D2 can still be earned on the slope they give, by error carried forward. These points lie so close to a straight line that a good line often runs through the end rows, $(0.100,\ 0.20)$ and $(0.500,\ 1.00)$: two points chosen from the table that the drawn line does pass through are points on the line, and earn D1. What loses it is a table point the line misses.

EXAMPLE RESPONSE

Two points on the line, $(0.15\,\mathrm{kg},\ 0.30\,\mathrm{s}^2)$ and $(0.45\,\mathrm{kg},\ 0.90\,\mathrm{s}^2)$: slope $= \dfrac{0.90 - 0.30}{0.45 - 0.15} = 2.0\,\mathrm{s}^2/\mathrm{kg}$.

MINIMUM SUFFICIENT Two points on the drawn line — read off it, or table points it passes through — and the slope computed from them as rise over run. **Not enough:** a slope with no points shown; **also not enough:** a table point that the drawn line does not pass through.

For an experimental value of k, with its unit, obtained from the slope through the relation that matches the axes. **Point D2**

Accept the relation that matches the axes — for the book's pair:

- T^2 against m: $k = 4\pi^2/\text{slope}$
- m against T^2: $k = 4\pi^2 \cdot \text{slope}$

Scoring Note. Accept 18 to 23 N/m; T against $\sqrt{m}$ uses $k = 4\pi^2/\text{slope}^2$, from the list under B2. The band is wide because the slope is read off a hand-drawn line: lines through these points run from about 1.9 to 2.1 s^2/kg, which gives k from about 19 to 21 N/m; a least-squares fit of the table gives 19.8, and the book 20 N/m. Each slip in the conversion lands far outside: the slope itself taken as k gives 2.0, 4π in place of $4\pi^2$ gives 6.3, and $4\pi^2 \cdot$ slope on the book's axes gives 79. **Error carried forward,** as in 4.3 D2 and 6.3 D2: where D1 was lost for the way the slope was found, D2 is judged on the relation alone, inside the band or out of it; where D1 was earned, the band decides. A value worked from a single row of the table uses no line and earns nothing here, even though every row lands inside the band. kg/s^2 is as good a unit as N/m.

EXAMPLE RESPONSE

$$k = \frac{4\pi^2}{\text{slope}} = \frac{39.5}{2.0\,\text{s}^2/\text{kg}} \approx 20\,\text{N/m}.$$

MINIMUM SUFFICIENT A value between 18 and 23 N/m, with the unit, that follows from the response's own slope through the relation for its axes — or, where D1 was lost, whatever that relation gives from the response's own slope. **Not enough:** 2.0, the slope itself; a value from one row of the table; a bare number with no unit.

Problem 7.4: Qualitative Quantitative Translation (QQT) — 8 rubric points

The problem. Two blocks are attached to identical horizontal springs of constant k on frictionless surfaces: Block 1 of mass M, Block 2 of mass $3M$. Both springs are compressed by the same distance A and the blocks are released from rest at the same instant; each then oscillates in simple harmonic motion. v_1 and v_2 are the two blocks' maximum speeds.

Part (a) *Indicate whether v_1 is greater than, less than or equal to v_2, and justify in terms of the energy of each block–spring system, with qualitative reasoning beyond referencing equations.*

For selecting $v_1 > v_2$. — Point A1

Scoring Note. The selection alone earns this point.

EXAMPLE RESPONSE

$v_1 > v_2$ is marked.

MINIMUM SUFFICIENT The $v_1 > v_2$ box marked, ticked or circled, or the choice written out unambiguously in words. **Not enough:** more than one box marked.

For what the two systems share: the same spring constant and the same compression, so the same energy stored at release — the energy each block carries as kinetic energy at equilibrium. — Point A2

Scoring Note. The box beside the book's solution names the two halves of this answer, and both are needed: the energy that is the same in both systems, and how the mass changes what that energy does. "The heavier block has more inertia, so it is slower" is the incomplete answer that box describes — it has no word on the energy — and does not earn A2. "The springs push with the same force over the same distance, so they do the same work on each block" is the same fact seen from the work side, and earns it.

EXAMPLE RESPONSE

Both springs have the same k and are compressed by the same A, so each system starts with the same spring energy, $\frac{1}{2}kA^2$. At equilibrium the spring has its natural length, and all of that energy has become the block's kinetic energy.

MINIMUM SUFFICIENT Same k and same compression, so the same stored energy — or the same work done by each spring — in both systems. **Not enough:** "the springs are identical" with nothing about the energy; **also not enough:** "the heavier block has more inertia".

For what differs: with the same kinetic energy at equilibrium, the block with three times the mass has to move more slowly. — Point A3

Scoring Note. Kinetic energy grows with the mass and with the square of the speed, so the same amount of it, carried by more mass, means less speed. A3 asks for that link between energy, mass and speed in words: "Block 2 is heavier, so it is slower" leaves the energy out and does not earn it. The stem wants reasoning beyond equations, so $v = \sqrt{2K/m}$ with no words earns neither A2 nor A3 — the same ruling as 4.4 (a) and 5.4 (a). A2 and A3 are marked independently.

EXAMPLE RESPONSE

Block 2 has three times the mass of Block 1 but the same kinetic energy at equilibrium. To carry the same kinetic energy, more mass has to move more slowly, so Block 2's maximum speed is the smaller one: $v_1 > v_2$.

MINIMUM SUFFICIENT One sentence: with the same kinetic energy, the larger mass means a smaller speed. **Not enough:** "Block 2 is heavier, so it is slower", with no energy in it; **also not enough:** $v = \sqrt{2K/m}$ with no words.

Part (b) *Starting with conservation of energy, derive the maximum speed $v_{\max}$ of a block of mass m on a spring of constant k that oscillates with amplitude A, in terms of m, k and A, beginning from a fundamental principle or a reference equation.*

For opening with conservation of energy, written as a statement before it is applied. — Point B1

Scoring Note. The stem names the principle, so the ruling of 7.2 B1 applies word for word. $v_{\max} = A\omega$ with $\omega = \sqrt{k/m}$ — the route the box beside this part warns against — reaches the right answer from a derived relation and earns B2 and B3, as a bare answer would, never B1.

EXAMPLE RESPONSE

Conservation of energy for the block–spring system: $K_i + U_{s,i} = K_f + U_{s,f}$.

MINIMUM SUFFICIENT $E_i = E_f$, the energies at the two positions set equal, or $\Delta K = \sum W$ — in any notation. **Not enough:** "energy is conserved" in words only; **also not enough:** $v_{\max} = A\omega$.

For the energies at release and at equilibrium: $\frac{1}{2}kA^2 = \frac{1}{2}mv_{\text{max}}^2$. **Point B2**

Scoring Note. At release the block is at rest; at equilibrium the spring has its natural length. As in 7.2 B2, a half dropped from one side costs this point.

EXAMPLE RESPONSE

$0 + \frac{1}{2}kA^2 = \frac{1}{2}mv_{\text{max}}^2 + 0$.

MINIMUM SUFFICIENT $\frac{1}{2}kA^2 = \frac{1}{2}mv_{\text{max}}^2$, in any arrangement. **Not enough:** a half on one side only; **also not enough:** kA for the spring's energy.

For the correct expression $v_{\text{max}} = A\sqrt{\frac{k}{m}}$. **Point B3**

Scoring Note. A bare correct expression carries B2 as well, but not B1. Error carried forward applies to a wrong term that is present: a half dropped from one side gives $A\sqrt{2k/m}$ or $A\sqrt{k/(2m)}$, either of which earns B3 and is then read in part (c).

EXAMPLE RESPONSE

$kA^2 = mv_{\text{max}}^2$, so $v_{\text{max}} = A\sqrt{\frac{k}{m}}$.

MINIMUM SUFFICIENT $v_{\text{max}} = A\sqrt{k/m}$ or an equivalent, with no working required — or what the response's own B2 equation gives. **Not enough:** $v_{\text{max}}^2 = kA^2/m$.

Part (c) *Indicate whether the expression from part (b) is consistent with the claim in part (a), and briefly justify by referring to specific terms in the derivation.*

For selecting *Consistent*. **Point C1**

Scoring Note. Award this point on the selection alone. Award *Not consistent* only where the response says in so many words that its own expression disagrees with its part (a) claim — the same ruling as 4.4 C1. As in every QQT of the series, the bare selection earns this point and nothing more; the reasoning point, C2, needs the expression.

EXAMPLE RESPONSE

Consistent is marked.

MINIMUM SUFFICIENT The *Consistent* box marked, ticked or circled, or the choice written out unambiguously in words — or *Not consistent* where the response says outright that its own expression contradicts its part (a) claim. **Not enough:** both boxes marked, or no choice at all.

For using the expression's dependence on the mass: m sits in the denominator under the square root, so with the same k and A the block of mass $3M$ has the smaller maximum speed. **Point C2**

Accept any of these:

- $v_{\text{max}} \propto 1/\sqrt{m}$, so the heavier block is the slower
- $v_2 = v_1/\sqrt{3}$, less than v_1
- m in the denominator, with k and A the same for both blocks, so Block 2 is the slower
- on a carried-forward expression, a correct reading of how its v_{max} changes with m, whatever that says about part (a)

Scoring Note. The expression has to be pointed at and used to order the two speeds, as in 2.8 C2: "it agrees with part (a)" does not do that, however true. Error carried forward, as in 4.4 C2 and 6.4 C2: read the response's own expression.

EXAMPLE RESPONSE

In $v_{\text{max}} = A\sqrt{k/m}$, k and A are the same for both blocks and m sits in the denominator under the root, so $v_{\text{max}} \propto 1/\sqrt{m}$: Block 2, with $3M$, reaches $1/\sqrt{3}$ of Block 1's speed. That is $v_1 > v_2$ — the claim of part (a).

MINIMUM SUFFICIENT A sentence pointing at m's place in the expression, or at $v_{\text{max}} \propto 1/\sqrt{m}$, and concluding that the heavier block is the slower — or, on a carried-forward expression, a sentence reading how its v_{max} changes with m. **Not enough:** "yes, consistent" with no reference to the expression.

Set B — Reinforcement

Problem 7.5 • 8 rubric points • a second Qualitative Quantitative Translation problem, about eighteen minutes, on the restoring force that defines simple harmonic motion — a supplement to Set A, not a paper of its own. Its tally sheet is the last page.

Problem 7.5: Qualitative Quantitative Translation (QQT) — 8 rubric points

The problem. A block of mass M on a frictionless horizontal surface, attached to a spring of constant k fixed to a wall, oscillates in simple harmonic motion with amplitude A about equilibrium at $x = 0$. It passes through $x_1 = A/4$ and, later, through $x_2 = 3A/4$; a_1 and a_2 are the magnitudes of its acceleration at those two positions.

Part (a) *Indicate whether a_1 is greater than, less than or equal to a_2, and justify in terms of the forces exerted on the block at each position, with qualitative reasoning beyond referencing equations.*

For selecting $a_1 < a_2$. **Point A1**

Scoring Note. The selection alone earns this point.

EXAMPLE RESPONSE

$a_1 < a_2$ is marked.

MINIMUM SUFFICIENT The $a_1 < a_2$ box marked, ticked or circled, or the choice written out unambiguously in words. **Not enough:** more than one box marked.

For the forces at both positions: the spring's force is the only horizontal force on the block — its weight and the normal force are vertical and balance, and there is no friction — so it is the whole of the net force at x_1 and at x_2 alike. **Point A2**

Scoring Note. The book sets this answer out as what stays the same and what changes, and this half is about the forces, which the stem asks for: one force, the spring's, decides the acceleration at both positions, acting on one block whose mass does not change. The book names the unchanged mass and spring constant here too, and a response may add them, but neither alone earns A2: "the mass is the same" says nothing about the forces. "The spring force is the only force on the block" passes over the weight and the normal force, but they cancel, and it is read as naming the net force: it earns A2. "The surface is frictionless" on its own is the start of the error the book names beside this part, the argument that the two accelerations are equal.

EXAMPLE RESPONSE

At both positions the block's weight and the normal force from the surface are vertical and cancel, and there is no friction, so the spring's force is the only horizontal force on it — the net force — at x_1 and at x_2 alike, acting on the same mass M.

MINIMUM SUFFICIENT The spring force named as the only horizontal force on the block — or as the net force — at both positions. **Not enough:** "the mass is the same" alone; **also not enough:** "the surface is frictionless" alone.

For what changes: the spring is stretched farther at x_2, so it pulls harder there — its force grows with the stretch — and the larger force gives the larger acceleration. **Point A3**

Scoring Note. The spring's force is proportional to the stretch, and at x_2 the stretch is three times that at x_1, so the force, and with it the acceleration, is three times as large; the factor is the model answer, not the threshold. A3 needs both links: the force larger at x_2 because of the greater stretch, and the larger force giving the larger acceleration. "In simple harmonic motion the acceleration is constant" is the error the book names beside this part. The stem asks for reasoning beyond equations, so $F = kx$ or $a = kx/M$ with no words earns neither A2 nor A3 — the same ruling as 7.4 (a). A2 and A3 are marked independently.

EXAMPLE RESPONSE

The spring's force grows in proportion to how far the spring is stretched. At $x_2 = 3A/4$ the spring is stretched three times as far as at $x_1 = A/4$, so it pulls back on the block three times as hard. Three times the force on the same mass means three times the acceleration: $a_1 < a_2$.

MINIMUM SUFFICIENT One sentence: the spring is stretched more at x_2, so its force is larger there, and the larger force gives the larger acceleration. **Not enough:** "the spring force is larger at x_2" with no reason, or with no link to the acceleration; **also not enough:** "x_2 is farther from equilibrium, so the acceleration is larger", which skips the force.

Part (b) *Starting with Newton's second law, derive the acceleration a of the block as a function of its displacement x from equilibrium, in terms of M, k and x, beginning from a fundamental principle or a reference equation.*

For opening with Newton's second law, written in general symbols before the spring's force goes in. **Point B1**

Scoring Note. The stem names the law, so it has to be on the page before it is applied — the opening rule of 5.2 B1 and 6.4 B1. A first line $-kx = Ma$ has applied the law and counts toward B2, but not B1. $a = -\omega^2 x$, quoted from the description of simple harmonic motion, reaches the right form from a derived relation and earns what a bare answer earns, B2 and B3, never B1.

EXAMPLE RESPONSE

Newton's second law for the block, horizontally: $F_{\text{net},x} = Ma$.

MINIMUM SUFFICIENT $\sum F = Ma$ or $F_{\text{net}} = Ma$, in any notation, written first. **Not enough:** the law named in words only; **also not enough:** $-kx = Ma$ with nothing above it, which counts toward B2 only.

For the spring's force as the whole net horizontal force, $F_{\text{net}} = -kx$, with x the displacement, not the amplitude. **Point B2**

Scoring Note. The spring's force is the only horizontal force — A2's fact — so it is the net force, and Hooke's law gives it from the displacement. The minus sign says the force points back toward equilibrium; a magnitude, kx, with the direction stated in words or left to the comparison of magnitudes the stem is about, also earns B2. kA in place of kx is the error the book names beside this part: it is the force at the turning points only, and it costs this point.

EXAMPLE RESPONSE

The spring's force is the only horizontal force on the block, so $F_{\text{net},x} = F_s = -kx$, and $-kx = Ma$.

MINIMUM SUFFICIENT $-kx$, or its magnitude kx, set equal to Ma as the net force. **Not enough:** kA in place of kx; **also not enough:** a friction or weight term added to the horizontal sum.

For the correct expression $a = -\dfrac{k}{M}\,x$. **Point B3**

Accept any of these forms:

- $a = -kx/M$, or $a = kx/M$ with the sign left out, or $|a| = k|x|/M$

Scoring Note. A bare correct expression carries B2 as well, but not B1. Parts (a) and (c) compare magnitudes, so the sign is not what this point checks: with or without the minus sign, kx/M earns B3. Error carried forward applies to a wrong term that is present in B2's equation — the spring's energy, $\frac{1}{2}kx^2$, written in place of its force, say, which gives $a = kx^2/(2M)$ and earns B3. It does not reach $a = kA/M$: the stem asks for a function of x, and an answer with no x in it is not one. Part (c) still reads whatever expression the response has.

EXAMPLE RESPONSE

$a = -\dfrac{k}{M}\,x$, so $|a| = \dfrac{k}{M}\,|x|$.

MINIMUM SUFFICIENT $a = -kx/M$, with or without the minus sign, with no working required — or what the response's own B2 equation gives, as a function of x. **Not enough:** $a = kA/M$, which is not a function of x.

Part (c) *Indicate whether the expression from part (b) is consistent with the claim in part (a), and briefly justify by referring to specific terms in the derivation.*

For selecting *Consistent*. **Point C1**

Scoring Note. The ruling of 7.4 C1: the selection alone, or *Not consistent* where the response says in so many words that its own expression contradicts its part (a) claim.

EXAMPLE RESPONSE

Consistent is marked.

MINIMUM SUFFICIENT The *Consistent* box marked, ticked or circled, or the choice written out unambiguously in words — or *Not consistent* where the response says outright that its own expression contradicts its part (a) claim. **Not enough:** both boxes marked, or no choice at all.

For using the expression's dependence on x: x sits in the numerator, so the size of the acceleration grows in proportion to the displacement, and $x_2 > x_1$ gives $a_2 > a_1$. **Point C2**

Accept any of these:

- $|a| \propto |x|$, so the point farther from equilibrium has the larger acceleration
- $a_2 = 3a_1$, from $x_2 = 3x_1$ in the expression
- on a carried-forward expression, a correct reading of how its a changes with x, whatever that says about part (a)

Scoring Note. The expression has to be pointed at and used to order the two accelerations: "yes, it agrees" does not do that, however true — the box beside the book's solution says as much. k and M are the same at both positions and need not be mentioned. Error carried forward, as in 7.4 C2: read the response's own expression. $a = kA/M$ has no x in it, and a response that reads it as giving the same acceleration at both positions earns C2, even though that reading contradicts part (a).

EXAMPLE RESPONSE

In $|a| = \dfrac{k}{M}|x|$, k and M do not change and x sits in the numerator, so the acceleration is proportional to the displacement. $x_2 = 3A/4$ is three times $x_1 = A/4$, so $a_2 = 3a_1$, and $a_1 < a_2$ — the claim of part (a).

MINIMUM SUFFICIENT A sentence pointing at x in the numerator, or at $|a| \propto |x|$, and concluding that the acceleration is larger at x_2 — or, on a carried-forward expression, a sentence reading how its a changes with x. **Not enough:** "yes, it agrees" with no reference to the expression.

Tally sheet • Set A

Problems 7.1–7.4, forty points — a complete free-response paper. One box per point, in order. Photocopy it, or keep one on the desk and write the student's name at the top. The short labels are reminders, not the rubric — the ruling for each point is on its own page above.

7.1 MR 10 pts

- ☐ **A1** F_g down, F_T up and to the left along the string, both labelled — no components, nothing else
- ☐ **A2** F_T visibly shorter than F_g
- ☐ **A3** opens with $E_i = E_f$ (or $\Delta K = \sum W$) in general symbols
- ☐ **A4** $MgL(1-\cos\theta_0) = \frac{1}{2}Mv_{\max}^2$ — h written out
- ☐ **A5** $v_{\max} = \sqrt{2gL(1-\cos\theta_0)}$, or ecf — not $\sqrt{2gh}$, not $\theta_0\sqrt{gL}$ alone
- ☐ **A6** 1.4 m/s with unit, or ecf — not 4.4–4.5 m/s from L put in for h here
- ☐ **A7** opens with $\sum F = Mv^2/L$ — not $\sum F = 0$
- ☐ **A8** $F_T = Mg(3-2\cos\theta_0)$, or ecf on $v_{\max}$ only — not Mg
- ☐ **B1** "Greater than"
- ☐ **B2** weaker pull back to the bottom (or less speed from the same drop) ⇒ slower swing ⇒ longer period, in words

7.2 TBR 12 pts

- ☐ **A1** $x = 0$: K up to the E line, no U_s bar
- ☐ **A2** $x = A/2$: U_s from $\frac{1}{8}$ to $\frac{3}{8}$ of E, K filling the rest to E
- ☐ **B1** opens with $E_i = E_f$ (or $\Delta K = \sum W$)
- ☐ **B2** $\frac{1}{2}kA^2 = \frac{1}{2}Mv_{\max}^2$
- ☐ **B3** $v_{\max} = A\sqrt{k/M}$, or ecf
- ☐ **B4** $\frac{1}{2}kA^2 = \frac{1}{2}Mv^2 + \frac{1}{2}k(A/2)^2$, squared right
- ☐ **B5** $v = (A/2)\sqrt{3k/M}$ in M, k, A, or ecf — not with the spring term left out
- ☐ **C1** E: labelled level line, $-A$ to $+A$
- ☐ **C2** U_s: labelled bowl, 0 at $x = 0$, meeting E at $\pm A$ — not a V
- ☐ **C3** K: labelled upturned bowl, E at $x = 0$, 0 at $\pm A$ — not an inverted V
- ☐ **D1** "Greater than" (or what *their own* curves give, a level K included)
- ☐ **D2** K curve higher at $A/4$ (or U_s lower), so faster — read off the curves; or what *their own* curves give

7.3 LAB 10 pts

- ☐ **A1** T for several known masses (or the stretch for several masses)
- ☐ **A2** a real uncertainty-reducing step (several oscillations ÷ their number, repeats), not just several masses
- ☐ **B1** a linear pair: T^2 and m, either way round, or another
- ☐ **B2** k from the slope, conversion matching its axes
- ☐ **C1** T^2 and m in the blanks (or another accepted pair)
- ☐ **C2** both axes: name and unit (s^2 for T^2); uniform scale where renumbered
- ☐ **C3** at least four of the five points in place
- ☐ **C4** one straight line through the points
- ☐ **D1** slope from two points *on the line*
- ☐ **D2** k from the slope via its axes; 18–23 N/m if D1 earned

7.4 QQT 8 pts

- ☐ **A1** $v_1 > v_2$
- ☐ **A2** same k, same A ⇒ same stored energy (or same work) — in words
- ☐ **A3** same K, larger mass ⇒ smaller speed — in words
- ☐ **B1** opens with $E_i = E_f$ — not $v_{\max} = A\omega$
- ☐ **B2** $\frac{1}{2}kA^2 = \frac{1}{2}mv_{\max}^2$
- ☐ **B3** $v_{\max} = A\sqrt{k/m}$, or ecf
- ☐ **C1** "Consistent" (or a contradiction it states)
- ☐ **C2** $v_{\max} \propto 1/\sqrt{m}$ from the expression, so $v_1 > v_2$ — or reads *their own* expression

SET A TOTAL

7.1 _____ / 10
7.2 _____ / 12
7.3 _____ / 10
7.4 _____ / 8

_____ / 40

Scoring key, Unit 7: Oscillations — Set A. Written for *AP Physics 1: Algebra-Based — Practice Workbook*, Chapter 7, problems 7.1–7.4. Original rubrics for original problems, set out in the format of published scoring guidelines. Not a College Board document and not endorsed by College Board. AP® is a registered trademark of College Board, which was not involved in producing this material.

Tally sheet • Set B

Problem 7.5, eight points — the chapter's reinforcement problem. Same arrangement as Set A: one box per point, in order, with short reminders rather than the rulings themselves.

7.5 QQT 8 pts

- ☐ **A1** $a_1 < a_2$
- ☐ **A2** the spring force is the only horizontal (net) force at both positions — in words
- ☐ **A3** stretched more at $x_2 \Rightarrow$ larger force $\Rightarrow$ larger a — in words
- ☐ **B1** opens with $\sum F = Ma$
- ☐ **B2** $-kx = Ma$ (or kx) — not kA
- ☐ **B3** $a = -kx/M$, sign optional, or ecf — not kA/M
- ☐ **C1** "Consistent" (or a contradiction it states)
- ☐ **C2** $|a| \propto |x|$ from the expression, so $a_2 > a_1$ — or reads *their own* expression

SET B TOTAL

7.5 _____ / 8

_____ / 8

The two sheets together come to forty-eight points, the chapter total. Only Set A is a complete paper; Set B adds a second QQT on the restoring force for a class ready to go past energy.

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