Scoring Key
Unit 8: Fluids

Forty-eight rubric points across the five free-response problems of Chapter 8 — and, at each one, the shortest answer that still earns it.

What this key is for

A suggested way to score problems 8.1–8.5 of the AP Physics 1 workbook, to help teachers grade them quickly and consistently. The format follows College Board's published scoring guidelines, but the points and thresholds are this book's own reading of the standard — another reader might draw a line differently.

The problems in this workbook are original, so there is no official rubric for them and this one is the book's own. **It is not a College Board document and is not endorsed by College Board.** AP® is a registered trademark of College Board, which was not involved in producing this material.

How to read one point

Every point carries four things, in this order.

For …	The condition. One sentence saying what the response must contain. The label on the right — Point A1 — names the point; the letter is the part of the problem.
Scoring Note	A ruling on an edge case: what two parts may be scored together, which errors are carried forward, what a bare final answer is worth.
Example Response	A full-credit answer. It is a model, not a threshold — a response can be much thinner than this and still earn the point.
Minimum sufficient	**The threshold.** The least a student can put on the page and still be given the point, plus, where it matters, the near-miss that sits just below it. This is the line published guidelines leave to the reader's judgement; here it is written down so that twenty-five papers can be graded in a quarter of an hour and graded the same way twice.

Three standing rules. A point is earned independently of those around it unless a Scoring Note says otherwise; an error made early and carried through consistently is scored as correct downstream — though leaving out a term the part is about, such as the cube's weight when the string's tension is found, or the height term in the pressure at the top of a rising pipe, answers a different question and is not carried forward; and a bare answer earns the points its Scoring Note names, never more.

What each problem is worth

The totals are not free choices. They are the point values the 2025 exam gives each question type — ten, twelve, ten and eight, forty on the paper. Chapter 8 has one problem of every type and a second Qualitative Quantitative Translation problem. The book groups them the way this key does: Set A, problems 8.1–8.4, is a complete ninety-minute free-response section; Set B is problem 8.5, a second QQT on density and draining tanks — about eighteen minutes, a supplement rather than a second paper.

PROBLEM	TYPE	WHAT IT TESTS	POINTS
Set A — Core			
8.1	MR	A cube held under a liquid by a string tied to the tank's floor; the force diagram, the buoyant force from the pressures on its faces, the tension and its value, and a denser cube	**10**
8.2	QQT	Water entering the narrow part of a level pipe; why its pressure drops, the pressure difference from Bernoulli's equation, and whether the two agree	**8**
8.3	LAB	A liquid's density from a gauge-pressure sensor and a meterstick; then ρ from the slope of gauge pressure against depth	**10**
8.4	TBR	A pipe that narrows and then climbs; Bernoulli bar charts, v_2 and P_3, speed and pressure along the pipe, and a claim that equal speeds mean equal pressures	**12**
		Set A total	**40**
Set B — Reinforcement			
8.5	QQT	Two open tanks of different liquids, filled to the same height; why they drain equally fast, that speed from Bernoulli's equation, and whether the two agree	**8**
		Set B total	**8**
		Chapter 8 total	**48**

MR Mathematical Routines • **TBR** Translation Between Representations • **LAB** Experimental Design and Analysis • **QQT** Qualitative Quantitative Translation.

Set A — Core

Problems 8.1–8.4 • 40 rubric points • one of each question type • a full ninety-minute free-response section. The tally sheet for this set is on the last page but one.

Problem 8.1: Mathematical Routines (MR) — 10 rubric points

The problem. A uniform solid cube of side s and mass M, its density $\rho_c = M/s^3$ less than the liquid's, ρ_L, is held fully under the liquid by a light string tied to the bottom of a large open tank, with its top face at depth d; everything is at rest. Part (b) puts in a denser cube of the same size, still less dense than the liquid, at the same depth.

Part (a) i *On the dot, draw and label the forces (not components) exerted on the cube, each as an arrow starting on the dot, with lengths that reflect their relative magnitudes.*

For the forces on the cube, each drawn from the dot and labelled, in the right directions — the buoyant force up, the cube's weight down and the string's tension down — and no others. **Point A1**

Scoring Note. The string is tied to the floor of the tank, so it pulls the cube down. A tension drawn upward — the error the book names beside this diagram, from picturing a string that holds the cube up — costs the point. The liquid's push is one force, the buoyant force: an arrow for "pressure" drawn beside it counts the liquid twice, the other error the book names here, and costs A1. Two arrows in its place — down on the top face, up on the bottom face — show the same push in two pieces and are accepted, as long as no buoyant-force arrow is drawn as well; the pushes on the four sides cancel in pairs and need not appear, and drawn as opposite pairs they cost nothing. Any clear labels will do: F_B or "buoyant", Mg, F_g or "weight", T or "string". Lengths are A2's business.

EXAMPLE RESPONSE

Three arrows from the dot: F_B straight up; T and Mg straight down.

MINIMUM SUFFICIENT Labelled arrows from the dot — buoyant force up, weight down, tension down, or the liquid's push as face arrows in place of F_B — and nothing else. **Not enough:** the tension drawn up; **also not enough:** an extra arrow, such as a pressure force beside F_B.

For the lengths: the buoyant-force arrow the longest, about as long as the tension and weight arrows together. **Point A2**

Scoring Note. The cube is in equilibrium, so $F_B = Mg + T$: laid end to end, the two downward arrows should come to the upward one's length, within about a seventh either way; a ruler settles a doubtful case. Nothing in part (a) i fixes which downward arrow is longer — $T/Mg = \rho_L/\rho_c - 1$ depends on the cube — so either may be; with part (a) iv's numbers the tension is the longer, 7.0 against 5.0 N, and a diagram drawn that way is just as good. In copies printed before the correction, the book's own diagram draws the buoyant arrow longer than each downward arrow but about a quarter short of the two together, and the sentence under it asks only that it be longer than each; a diagram drawn to that standard does not earn A2, and the lower end of the band sits about halfway to it. The corrected diagram is listed on the workbook's errata page, under page 336. With the liquid drawn as two face arrows, the upward one has to match the other three together, and side pushes, if drawn, should be about equal in pairs. A2 reads lengths only; the directions were A1's.

EXAMPLE RESPONSE

T and Mg drawn about 1.5 cm and 1.8 cm long, and F_B 3.3 cm, as long as the two together.

MINIMUM SUFFICIENT An upward arrow visibly the longest and as long as the two downward arrows end to end, within about a seventh. **Not enough:** an upward arrow longer than each downward one but visibly short of the two together, as in the diagram of copies printed before the correction; **also not enough:** a weight arrow as long as the buoyant one.

Part (a) ii *Starting with pressure as a function of depth, derive the net upward force exerted by the liquid on the cube, in terms of ρ_L, s and g.*

For opening with pressure as a function of depth, written in general symbols before it is applied to the cube's faces. **Point A3**

Accept any of these openings:

- $P = P_0 + \rho gh$, with ρ the liquid's density
- the gauge form $P = \rho gh$, or $\Delta P = \rho g\,\Delta h$

Scoring Note. The stem names the starting point, so the series' opening rule applies: the relation goes on the page before it is used. A first line that already reads $P_{\text{top}} = P_0 + \rho_L g d$ has applied it correctly and counts toward A4, but not this point. Archimedes' principle, $F_B = \rho_L V g$, is a different starting point: it reaches the right force by a road the stem did not ask for, and earns what a bare answer earns, A4, never A3.

EXAMPLE RESPONSE

In a liquid at rest, the pressure at depth h is $P = P_0 + \rho_L gh$.

MINIMUM SUFFICIENT $P = P_0 + \rho gh$, or its gauge form, in general symbols before either face is named. **Not enough:** $F_B = \rho V g$ as the first line; **also not enough:** the relation in words only.

For the correct expression $F_B = \rho_L g s^3$. **Point A4**

Scoring Note. The bottom face is deeper by s, so its pressure exceeds the top face's by $\rho_L g s$, over an area s^2; the pushes on the four sides cancel in pairs. The depth d has to cancel: an answer that keeps it — the error the book names beside this part — does not earn A4, and neither does $\rho_L g(d+s)s^2$, the push on the bottom face alone, or $\rho_L g s$, a pressure difference never multiplied by the area. A bare correct expression earns A4 but not A3, and so does the route through Archimedes' principle.

EXAMPLE RESPONSE

$P_{\text{top}} = P_0 + \rho_L g d$ and $P_{\text{bottom}} = P_0 + \rho_L g(d+s)$. The net upward force is $F_B = (P_{\text{bottom}} - P_{\text{top}})\, s^2 = (\rho_L g s)\, s^2 = \rho_L g s^3$.

MINIMUM SUFFICIENT $F_B = \rho_L g s^3$, with no working required. **Not enough:** an expression that still contains d; **also not enough:** $\rho_L g s$, with no area.

Part (a) iii *Using part (a) ii and Newton's second law for the cube in equilibrium, derive the tension T in the string, in terms of M, ρ_L, s and g.*

For opening with Newton's second law in general symbols, with the acceleration zero, before the cube's forces go in. **Point A5**

Accept any of these openings:

- $\sum F = ma$ with $a = 0$, or $\sum F_y = 0$
- $\sum F_{\text{up}} = \sum F_{\text{down}}$ for an object at rest

Scoring Note. The stem names the law, so the opening rule of A3 applies again. A first line $F_B - Mg - T = 0$ has applied the law and counts toward A6, but not A5. "The forces balance" in words alone is not an equation and does not earn it.

EXAMPLE RESPONSE

The cube is in static equilibrium: $\sum F_y = ma_y = 0$.

MINIMUM SUFFICIENT $\sum F = ma$ with $a = 0$, or $\sum F = 0$, in any notation, before F_B, Mg and T are put in. **Not enough:** $F_B - Mg - T = 0$ with nothing above it; **also not enough:** the law in words only.

For the correct expression $T = \rho_L g s^3 - Mg$. **Point A6**

Scoring Note. With up positive, the weight and the tension both point down and enter with the same sign. Tension taken as upward, $F_B + T - Mg = 0$ — the error the book names beside this part — gives $T = Mg - \rho_L g s^3$, a negative tension for a cube less dense than the liquid, and does not earn A6. Error carried forward from A4 applies to a wrong buoyant force that is present, put in correctly here. It does not apply to the cube's weight left out, $T = \rho_L g s^3$: that is page 1's exception, a cube with no weight. Nor does it apply to a buoyant force written with the cube's own density, $\rho_c g s^3$, or simply as Mg: that makes the tension zero for any cube, a result the wrong principle fixes in advance, and A7 refuses the same 0 N. A bare correct expression earns A6 but not A5.

EXAMPLE RESPONSE

$F_B - Mg - T = 0$, so $T = F_B - Mg = \rho_L g s^3 - Mg$.

MINIMUM SUFFICIENT $T = \rho_L g s^3 - Mg$ or an equivalent, with no working required — or what the response's own buoyant force gives, with the weight subtracted, unless that force is the cube's own weight. **Not enough:** $T = Mg - \rho_L g s^3$; **also not enough:** $T = \rho_L g s^3$, or $T = 0$.

Part (a) iv *Calculate T for $M = 0.50\,\text{kg}$, $s = 0.10\,\text{m}$ and $\rho_L = 1200\,\text{kg/m}^3$.*

For $T = 7.0\,\text{N}$, with its unit — or $6.9\,\text{N}$, from $g = 9.8\,\text{m/s}^2$. **Point A7**

Scoring Note. Copies printed before the correction give no value of g in this problem; the line "Use $g \approx 10\,\text{m/s}^2$" is listed on the workbook's errata page, under pages 330 and 332. A student working from such a copy who takes $9.8\,\text{m/s}^2$ from the reference sheet gets $6.9\,\text{N}$, and that value earns A7 exactly as 7.0 does. Error carried forward from part (a) iii applies within page 1's limits: this part checks the substitution, so the value the response's own expression gives earns A7. Four values are worth knowing on sight, given here for $g = 10$; with 9.8, from a copy printed before the correction, the first three become -6.9, 113 and $11.8\,\text{N}$. $-7.0\,\text{N}$, from the tension taken as upward, is carried, and so is the same number without its sign. $115\,\text{N}$, from an (a) ii or (a) iii with s^2 in place of s^3, is carried; the same number from a correct expression cubed wrongly here is not. $12\,\text{N}$ is the buoyant force alone, the weight left out, and is not carried. $0\,\text{N}$ comes from putting the cube's own density into the buoyant force, which makes it equal the weight whatever the numbers: a result the wrong principle fixes in advance, not carried, as A6 says.

EXAMPLE RESPONSE

$F_B = (1200)(10)(1.0 \times 10^{-3}) = 12\,\text{N}$ and $Mg = (0.50)(10) = 5.0\,\text{N}$, so $T = 12 - 5.0 = 7.0\,\text{N}$.

MINIMUM SUFFICIENT $7.0\,\text{N}$ or $6.9\,\text{N}$ with a unit — or the value the response's own expression gives, unless that expression leaves out the weight. **Not enough:** $12\,\text{N}$, or 11.8 with $g = 9.8$; $0\,\text{N}$; a bare number with no unit.

Part (b) *A cube of the same size but greater mass M', still less dense than the liquid, replaces the first at the same depth. Indicate whether the new tension T' is greater than, less than or equal to T, and justify with qualitative reasoning beyond derivations or expressions.*

For selecting *Less than*. **Point B1**

Scoring Note. The selection alone earns this point, whatever is written beside it.

EXAMPLE RESPONSE

Less than is marked.

MINIMUM SUFFICIENT The *Less than* box marked, ticked or circled, or the choice written out unambiguously in words. **Not enough:** two boxes marked, or no choice at all.

For the buoyant force unchanged, and why: the new cube has the same volume, so it displaces as much of the same liquid as the first. **Point B2**

Scoring Note. The buoyant force depends on the liquid and on the volume pushed aside, not on what the cube is made of. A larger buoyant force for the denser cube is the error the book names beside this part, and it does not earn B2. The reason can be short — "same size", "same volume in the same liquid", "it does not depend on the cube's mass" — and pointing at $\rho_L g s^3$ to say that neither ρ_L nor s has changed is a reason. A bare "the buoyant force is the same" is a claim with nothing behind it, and the stem asks for reasoning.

EXAMPLE RESPONSE

The new cube is the same size and is still fully under the same liquid, so it displaces the same volume of it: the buoyant force on it is the same as on the first.

MINIMUM SUFFICIENT The buoyant force is the same, with a reason: the same volume, or size, in the same liquid. **Not enough:** a larger buoyant force because the cube is denser; **also not enough:** "the buoyant force is the same" with no reason.

For the balance: the unchanged upward force must still equal the two downward forces together, so a heavier cube leaves less for the string to supply. **Point B3**

Scoring Note. The stem asks for reasoning beyond derivations, so $T' = \rho_L g s^3 - M'g < T$ does not earn B3 on its own, correct as it is: the algebra may support the argument, but it may not be the argument, the same ruling as 6.1 B2. What B3 needs is the link between the fixed buoyant force and the way it is shared between weight and tension; "the cube is heavier, so the tension is less" skips it. B2 and B3 are marked independently.

EXAMPLE RESPONSE

The cube is still at rest, so its weight and the string's pull together must equal the same buoyant force. The new cube's weight is larger and takes up more of that force, so the string has less to supply: $T' < T$.

MINIMUM SUFFICIENT One sentence: the buoyant force is fixed and must equal weight plus tension, so a larger weight means a smaller tension. **Not enough:** $T' = \rho_L g s^3 - M'g$ with no words; **also not enough:** "heavier, so less tension", with no balance.

Problem 8.2: Qualitative Quantitative Translation (QQT) — 8 rubric points

The problem. Water of density ρ flows steadily through a level pipe that narrows smoothly from cross-sectional area A_1, where its speed is v_1 and its pressure P_1, to $A_2 < A_1$, where they are v_2 and P_2.

Part (a) *Indicate whether P_2 is greater than, less than or equal to P_1, and justify in terms of the forces exerted on a small volume of water moving from region 1 into region 2, with qualitative reasoning beyond referencing equations.*

For selecting *Less than*. — Point A1

Scoring Note. The selection alone earns this point.

EXAMPLE RESPONSE

Less than is marked.

MINIMUM SUFFICIENT The *Less than* box marked, ticked or circled, or the choice written out unambiguously in words. **Not enough:** more than one box marked.

For what changes, tied to the narrowing: the water moves faster where the pipe is narrower. — Point A2

Scoring Note. The narrowing is what changes, and continuity is what turns it into a speed: the same volume each second through a smaller section means faster water. Naming the narrowing as the place or as the cause is enough — "it speeds up in the narrower section" and "narrower, so faster" both earn A2, and the volume-per-second reason need not be written. "The water speeds up in region 2" does not: "region 2" is only a label, and the point is for naming the narrowing. The force argument of A3 stands on this step but is marked on its own.

EXAMPLE RESPONSE

Each second the same volume of water passes every point of the pipe. Region 2 is narrower, so the water there has to move faster to carry that volume through the smaller area: $v_2 > v_1$.

MINIMUM SUFFICIENT Faster flow with the narrowing named — "it speeds up in the narrower section", or "narrower, so faster". **Not enough:** "the water speeds up in region 2", with nothing on the narrowing.

For the forces: a volume of water that speeds up as it enters region 2 needs a net force pointing downstream; the only horizontal forces on it are the pushes of the water behind and ahead, so the push from behind is the larger and the pressure upstream is the higher. — Point A3

Scoring Note. The stem fixes the route — forces on the moving water, not the equation. "Faster flow means lower pressure", quoted from Bernoulli's equation, is the conclusion of part (b), not reasoning about forces, and does not earn A3: the series' rule for a justification whose route the stem names, as at 3.1 B3. The other answer to look for is the one the book names beside this part: the narrow section "squeezes" the water and raises its pressure, which would push the water backward just as it speeds up. The chain needs all three links: speeding up, a net force downstream, a larger push from the upstream side.

EXAMPLE RESPONSE

As the water moves into region 2 it speeds up, so the net force on it points downstream. The only horizontal forces on it come from the water around it: the water behind pushes it forward and the water ahead pushes it back. For the net force to point forward, the push from behind must be the stronger, so the pressure behind it, P_1, is greater than the pressure ahead, P_2.

MINIMUM SUFFICIENT One sentence: the water speeds up, so the net force on it points forward, so the pressure behind it (P_1) is greater than the pressure ahead (P_2). **Not enough:** "faster flow, lower pressure", from Bernoulli's equation; **also not enough:** the narrow part squeezing the water to a higher pressure.

Part (b) *Starting with Bernoulli's equation, derive $P_1 - P_2$ in terms of ρ, v_1, A_1 and A_2.*

For opening with Bernoulli's equation in general symbols, before continuity goes in. **Point B1**

Accept any of these openings:

- $P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$, or $P + \frac{1}{2}\rho v^2 + \rho g y$ constant along the flow
- the same with the height terms left out, as the level pipe allows: $P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$

Scoring Note. The stem names the equation, so the opening rule applies. Height terms left out from the first line are accepted, the series' ruling for terms that drop out (3.2 B1): the pipe is level. A first line with v_2 already written as $A_1 v_1 / A_2$ has applied the equation and counts toward B2 and B3, but not B1.

EXAMPLE RESPONSE

Bernoulli's equation along the flow: $P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$.

MINIMUM SUFFICIENT Bernoulli's equation between the two regions in general symbols, with or without the height terms. **Not enough:** the equation named in words only.

For continuity: $A_1 v_1 = A_2 v_2$, so $v_2 = (A_1/A_2)\, v_1$, used to remove v_2. **Point B2**

Scoring Note. The smaller area goes with the larger speed. $v_2 = (A_2/A_1)\, v_1$, the relation turned upside down, is the error the book names beside this part; it costs B2, and only B2, since B3 reads the response's own continuity. Cancelling the height terms is not scored apart: the pipe is level.

EXAMPLE RESPONSE

$A_1 v_1 = A_2 v_2$, so $v_2 = \dfrac{A_1}{A_2}\, v_1$.

MINIMUM SUFFICIENT $v_2 = A_1 v_1 / A_2$, written or substituted. **Not enough:** $v_2 = A_2 v_1 / A_1$.

For the correct expression $P_1 - P_2 = \frac{1}{2}\rho v_1^2 \left[\left(\frac{A_1}{A_2} \right)^2 - 1 \right]$. **Point B3**

Accept any equivalent form:

- $\dfrac{\rho v_1^2 \left(A_1^2 - A_2^2\right)}{2A_2^2}$, or $\frac{1}{2}\rho \left[\left(\frac{A_1 v_1}{A_2} \right)^2 - v_1^2 \right]$

Scoring Note. A bare correct expression carries B2 as well — the areas can only have come from continuity — but not B1. Error carried forward applies to the upside-down continuity, which gives $\frac{1}{2}\rho v_1^2 \left[(A_2/A_1)^2 - 1 \right]$; carried through correctly, it earns B3. Stopping at $\frac{1}{2}\rho \left(v_2^2 - v_1^2 \right)$, the other error the book names here, does not earn it: v_2 is not among the quantities allowed.

EXAMPLE RESPONSE

$P_1 - P_2 = \frac{1}{2}\rho \left(v_2^2 - v_1^2 \right) = \frac{1}{2}\rho \left[\left(\frac{A_1}{A_2}\, v_1 \right)^2 - v_1^2 \right] = \frac{1}{2}\rho v_1^2 \left[\left(\frac{A_1}{A_2} \right)^2 - 1 \right]$.

MINIMUM SUFFICIENT $\frac{1}{2}\rho v_1^2 \left[(A_1/A_2)^2 - 1 \right]$ or an equivalent, with no working required — or what the response's own continuity gives. **Not enough:** an answer that still contains v_2.

Part (c) *Indicate whether the expression from part (b) is consistent with the claim in part (a), and briefly justify by referring to specific terms of the derivation.*

For selecting *Consistent*. **Point C1**

Scoring Note. Award this point on the selection alone. Award *Not consistent* only where the response says in so many words that its own expression disagrees with its part (a) claim — the ruling of 4.4 C1 and 6.4 C1. A bare "consistent" earns this point and nothing more, as in every QQT of the series.

EXAMPLE RESPONSE

Consistent is marked.

MINIMUM SUFFICIENT The *Consistent* box marked, ticked or circled, or the choice written out unambiguously in words — or *Not consistent* where the response says outright that its own expression contradicts its part (a) claim. **Not enough:** both boxes marked, or no choice at all.

For reading the sign off the expression: $A_1 > A_2$, so $(A_1/A_2)^2 - 1$ is positive; with $\frac{1}{2}\rho v_1^2$ positive too, $P_1 - P_2 > 0$, and $P_2 < P_1$. **Point C2**

Accept any of these:

- $A_1/A_2 > 1$, so the bracket, and with it $P_1 - P_2$, is positive
- $A_1^2 > A_2^2$ in the numerator of the equivalent form
- on a carried-forward expression, a correct reading of its sign, whatever that says about part (a)
- for $\frac{1}{2}\rho\,(v_2^2 - v_1^2)$, left with v_2 in it, that $v_2 > v_1$ makes it positive

Scoring Note. The term that decides the sign has to be pointed at and used — the book's tip beside this part says as much — as in 2.8 C2 and 6.4 C2: "it agrees with part (a)" does not do that, however true. Error carried forward, as in 4.4 C2 and 6.4 C2: read the response's own expression. The upside-down continuity makes the bracket negative; a response that says so, and that P_2 would then be the larger, earns C2.

EXAMPLE RESPONSE

In $P_1 - P_2 = \frac{1}{2}\rho v_1^2\left[(A_1/A_2)^2 - 1\right]$, $\frac{1}{2}\rho v_1^2$ is positive, and $A_2 < A_1$ makes $A_1/A_2 > 1$, so the bracket is positive. Then $P_1 - P_2 > 0$: P_2 is less than P_1, the claim of part (a).

MINIMUM SUFFICIENT A sentence pointing at $(A_1/A_2)^2 - 1$, or at $A_1 > A_2$ inside it, and concluding that $P_1 > P_2$ — or, on a carried-forward expression, a sentence reading its sign. **Not enough:** "yes, consistent" with no reference to the expression.

Problem 8.3: Experimental Design and Analysis (LAB) — 10 rubric points

The problem. Students find the density ρ of an unknown liquid. Equipment: a tall open container of the liquid, a sensor that reads gauge pressure, and a meterstick; the sensor's tip can be lowered to a depth h below the surface. A second experiment, with the same kind of equipment and another liquid, gives the gauge pressure P_g at five depths, each the mean of three readings: 490 Pa at 0.04 m, 960 at 0.08, 1420 at 0.12, 1900 at 0.16 and 2360 at 0.20 m.

Part (a) *Describe a procedure for collecting the data needed to determine ρ, including steps that reduce experimental uncertainty.*

For a procedure that reads the gauge pressure with the sensor's tip at several depths, each depth measured with the meterstick. — **Point A1**

Scoring Note. Gauge pressure grows in proportion to depth, $P_g = \rho g h$, so pairs of depth and pressure are the data. Part (b) asks for a graph, so one depth, however often its reading is repeated, fixes ρ in principle but leaves nothing to graph and does not earn A1 — the ruling of 4.3 A1 and 6.3 A1. The book names the vague version, "put the sensor in the liquid", as the error here: what is varied and what is read have to be said.

EXAMPLE RESPONSE

Lower the sensor until its tip is 0.04 m below the surface, measured with the meterstick from the surface down to the tip; hold it still and record the gauge pressure. Repeat at 0.08, 0.12, 0.16 and 0.20 m.

MINIMUM SUFFICIENT The gauge pressure read at several depths, each depth measured with the meterstick. **Not enough:** a reading at one depth only; **also not enough:** "put the sensor in the liquid and read it".

For a genuine uncertainty-reducing step, stated as an action. — **Point A2**

Accept any one of these:

- several readings at each depth, averaged
- every depth measured to the same point, the sensor's tip, from the liquid's surface
- the depths spread over as much of the container as possible, said in so many words

Scoring Note. Repetition alone earns this point, with or without the word "average". The book's own procedure measures every depth from the surface to the tip of the sensor, and this key counts that as an uncertainty step too: measuring sometimes to the tip and sometimes to the top of the probe is a real source of error. The several depths that A1 requires cannot be counted a second time, the same ruling as 6.3 A2; a stated spread can. "Be careful" and "measure accurately" are intentions, not actions.

EXAMPLE RESPONSE

Take three readings at each depth and average them, keeping the sensor still and clear of the walls and the bottom, and measure every depth from the surface to the sensor's tip.

MINIMUM SUFFICIENT One named action that reduces uncertainty — repeated, averaged readings count on their own, and so does measuring every depth to the sensor's tip. **Not enough:** "measure carefully"; **also not enough:** "use several depths", which A1 has already paid for.

Part (b) *Describe how the data could be graphed, and how the graph would be analysed to find ρ.*

For a pair of quantities that $P_g = \rho g h$ makes linear, with ρ in the slope. — **Point B1**

Accept any such pair:

- P_g and h, either way round
- the absolute pressure, $P_0 + P_g$, against h; or P_g against gh

Scoring Note. Naming a pair the relation makes linear is enough for B1, as in every LAB of this series. A pair that keeps ρ out of the slope — P_g against $1/h$, or P_g/h against h — does not earn it.

EXAMPLE RESPONSE

Plot P_g on the vertical axis against h on the horizontal axis. Since $P_g = \rho g h$, the points fall on a straight line through the origin.

MINIMUM SUFFICIENT P_g and h named as the two axes, either way round — or another pair the relation makes linear. **Not enough:** a pair that leaves ρ out of the slope.

For showing how ρ comes out of the slope, with the conversion right for the axes chosen. **Point B2**

Accept the relation that matches the response's axes:

- P_g, or the absolute pressure, against h: slope $= \rho g$, so $\rho =$ slope$/g$
- h against P_g: slope $= 1/(\rho g)$, so $\rho = 1/(g \cdot \text{slope})$
- P_g against gh: $\rho =$ slope

Scoring Note. Error carried forward from B1 applies to the choice of axes. "The slope is the density" on the book's axes leaves g inside it — one of the errors the book names beside this part — and does not earn B2. The other, writing $P_g = P_0 + \rho g h$ for a sensor that already leaves out the air's pressure, misplaces the intercept, not the slope, and does not cost this point.

EXAMPLE RESPONSE

Draw a straight best-fit line. Its slope is ρg, so $\rho = \dfrac{\text{slope}}{g}$.

MINIMUM SUFFICIENT A correct written relation between the slope and ρ for the chosen axes. **Not enough:** "the slope is ρ" on axes of P_g against h; "use the slope to find ρ".

Part (c) i *For the second experiment's data, indicate two quantities that, when graphed, give a straight line whose slope yields ρ.*

For naming P_g as the vertical-axis quantity and h as the horizontal-axis quantity in the blanks. **Point C1**

Scoring Note. Accept the reverse assignment and pairs that differ only by a constant factor, such as P_g against gh, and carry the choice forward through C2 to D2, as in 4.3 C1 and 6.3 C1.

EXAMPLE RESPONSE

Vertical axis: P_g, in Pa. Horizontal axis: h, in m.

MINIMUM SUFFICIENT P_g and h in the two blanks, either way round — or another pair the note accepts. **Not enough:** P_g and $1/h$.

Part (c) ii *Plot the data, label the axes with names, units and a scale, and draw a straight best-fit line.*

For both axes labelled with the quantity and its unit — for the book's pair, P_g in Pa and h in m — and a uniform scale on any axis the student renumbered. **Point C2**

Scoring Note. The grid printed under this part comes numbered, 0 to 0.24 across and 0 to 2800 up, and those numbers fit the book's pair only the book's way round. On the printed scale this point is for names and units; a response that turns the axes round has to renumber them, and a renumbered axis needs a uniform scale. A unit that contradicts the printed numbers — cm on the axis that runs to 0.24, kPa on the one that runs to 2800 — is the slip to look for.

EXAMPLE RESPONSE

Vertical axis labelled P_g (Pa); horizontal axis labelled h (m).

MINIMUM SUFFICIENT Both axes carrying a name and a unit, and a uniform scale on any axis the student renumbered. **Not enough:** names without units, or units without names; a unit that contradicts the printed numbers.

For the five points plotted, at least four of them in the right places for the quantities on the axes. **Point C3**

Scoring Note. On the printed grid each small square is 0.02 m across and 200 Pa up, and every depth falls on a grid line, so a point counts as in place within about half a small square, 100 Pa, of its pressure. One point out of place still earns C3; two or more lose it.

EXAMPLE RESPONSE

Points at $(0.04,\ 490)$, $(0.08,\ 960)$, $(0.12,\ 1420)$, $(0.16,\ 1900)$ and $(0.20,\ 2360)$, with h across and P_g up.

MINIMUM SUFFICIENT Five points plotted, at least four of them within about half a small square of where they belong for the axes as labelled. **Not enough:** two or more clearly misplaced points.

For one straight best-fit line drawn through the data. **Point C4**

Scoring Note. Straight, and running through the points rather than beside them, is the whole of this point: a dot-to-dot chain — the error the book names beside this graph — does not earn it. The relation puts the line through the origin, and the book's own line, $P_g = 11\,700\,h + 22$, misses it by only 22 Pa, but the student's line need not pass through it.

EXAMPLE RESPONSE

A single straight line through the scatter, close to $P_g = 11\,700\,h$, with points on both sides of it.

MINIMUM SUFFICIENT One straight line through the plotted points, with points on both sides of it. **Not enough:** a dot-to-dot chain or a curve.

Part (d) *Using the best-fit line, calculate an experimental value of ρ, showing how the slope was found from two points on the line — not data points.*

For a slope worked out from two points read off the drawn line, with both points identified. **Point D1**

Scoring Note. Two data points in place of two points on the line lose D1 — the stem rules them out by name — but D2 can still be earned on the slope they give, by error carried forward. If the two points chosen happen to lie on the line as well as in the table, award D1.

EXAMPLE RESPONSE

Two points on the line, $(0.05,\ 607)$ and $(0.19,\ 2245)$: slope $= \dfrac{2245 - 607}{0.19 - 0.05} = 1.17 \times 10^4\ \text{Pa/m}$.

MINIMUM SUFFICIENT Two coordinate pairs stated as read from the line, and the slope computed from them as rise over run. **Not enough:** a slope with no points shown; **also not enough:** two points that are plainly plotted data.

For an experimental value of ρ, with its unit, obtained from the slope through the relation that matches the axes. **Point D2**

Accept the relation that matches the axes — for the book's pair:

- P_g against h: $\rho = \text{slope}/g$
- h against P_g: $\rho = 1/(g \cdot \text{slope})$

Scoring Note. Accept $1.05 \times 10^3\ \text{kg/m}^3$ to $1.35 \times 10^3\ \text{kg/m}^3$, with g taken as 10 or $9.8\ \text{m/s}^2$; the other pairs accepted in C1 use their own relations — $\rho =$ slope for P_g against gh. Copies printed before the correction give no value of g in this problem — the line "Use $g \approx 10\ \text{m/s}^2$" is listed on the workbook's errata page, under pages 330 and 332 — and 9.8 from the reference sheet gives about $1.19 \times 10^3\ \text{kg/m}^3$, as good as the book's 1.17. The band is wide because the slope is read off a hand-drawn line: lines through these points run from about 11 500 to 12 000 Pa/m, and a least-squares fit gives 11 700, the book's value. The slip the book names lands far outside: 1.17×10^4, the slope with g never divided out. **Error carried forward,** as in 6.3 D2: where D1 was lost for the way the slope was found, D2 is judged on the relation alone, inside the band or out of it; where D1 was earned, the band decides. A value worked from a single row of the table uses no line and earns nothing here, even though every row lands inside the band.

EXAMPLE RESPONSE

$$\rho = \frac{\text{slope}}{g} = \frac{1.17 \times 10^4\ \text{Pa/m}}{10\ \text{m/s}^2} = 1.17 \times 10^3\ \text{kg/m}^3.$$

MINIMUM SUFFICIENT A value between 1.05×10^3 and $1.35 \times 10^3\ \text{kg/m}^3$, with the unit, that follows from the response's own slope through the relation for its axes — or, where D1 was lost, whatever that relation gives from the response's own slope. **Not enough:** 1.17×10^4, the slope itself; a value from one row of the table; a bare number with no unit.

Problem 8.4: Translation Between Representations (TBR) — 12 rubric points

The problem. Water of density ρ flows steadily through a pipe. Section 1, at ground level, has area $2A$ and speed v_1; the pipe narrows to area A for Section 2, still at ground level, speed v_2; then it turns upward and keeps area A to Section 3, at height h, speed v_3. The pressures are P_1, P_2 and P_3. The printed bar chart for Section 1 shows P at 8 units, $\frac{1}{2}\rho v^2$ at 1 unit and $\rho g y$ as a line on zero; every empty outline at Sections 2 and 3 is drawn to 9 units, the Section 1 total. The stem gives no value for h.

Part (a) *Complete the bar charts of P, $\frac{1}{2}\rho v^2$ and $\rho g y$ at Sections 2 and 3, bars starting on the zero line, any zero quantity drawn as a line on it, heights consistent with the scale.*

For Section 2: a $\frac{1}{2}\rho v^2$ bar about four times as tall as Section 1's, and no $\rho g y$ bar. — Point A1

Scoring Note. Halving the area doubles the speed, and the kinetic term goes as the square of the speed, so it grows fourfold, from 1 unit to 4. On the printed page Section 1's kinetic bar, one unit, is 4.2 mm tall and each outline 37.8 mm; a ruler settles a doubtful case. A bar twice Section 1's, from doubling the speed without squaring it, is the slip to look for. Accept three to five times Section 1's bar, 12.6 to 21 mm: the lower end is halfway to that slip, and no error the book names lies above. Section 2 is still at ground level, so $\rho g y$ is a line on zero or nothing at all — a zero left blank costs nothing — but a bar of any height costs the point. The pressure bar is judged through the total, at A3.

EXAMPLE RESPONSE

$\frac{1}{2}\rho v^2$ at 4 units, about 17 mm; $\rho g y$ a line on zero; P at 5 units, about 21 mm.

MINIMUM SUFFICIENT A kinetic bar three to five times as tall as Section 1's, and no $\rho g y$ bar. **Not enough:** a kinetic bar twice Section 1's, or shorter; **also not enough:** a $\rho g y$ bar at Section 2.

For Section 3: a $\frac{1}{2}\rho v^2$ bar level with the response's own Section 2 kinetic bar, and a $\rho g y$ bar above zero. — Point A2

Scoring Note. Sections 2 and 3 have the same area, so the water keeps its speed and its kinetic term. A shorter kinetic bar at Section 3, drawn because the water has climbed, is the error the book names beside this chart. Read the kinetic bar against the response's own Section 2 bar, within about half a unit, 2 mm: a mistake at Section 2 has already cost A1 and is not charged again here. Section 3 is at height h, so $\rho g y$ is positive. The stem gives no value of h, and the book's solution says the split between this bar and the pressure bar is not fixed: any visible $\rho g y$ bar is accepted. Whether the three bars add up is A3's business.

EXAMPLE RESPONSE

$\frac{1}{2}\rho v^2$ at 4 units, level with Section 2's; $\rho g y$ at, say, 2 units and P at 3, the split the book's own sketch uses.

MINIMUM SUFFICIENT A kinetic bar level with the response's own Section 2 kinetic bar, and a visible $\rho g y$ bar. **Not enough:** a Section 3 kinetic bar visibly shorter than Section 2's; **also not enough:** $\rho g y$ left at zero.

For Bernoulli's sum on both charts: at Section 2 and at Section 3 the three bars together come to about the outline's height, the 9 units of Section 1. — Point A3

Scoring Note. $P + \frac{1}{2}\rho v^2 + \rho g y$ is the same everywhere along the flow. This point is for the total, and it is earned independently of A1 and A2, as in 4.2 A3 and 6.2 A3: a kinetic bar of 2 units at Section 2 with a pressure bar of 7 still adds up, and earns A3 alone. At Section 3 the pressure bar is what is left over. A $\rho g y$ bar added beside pressure and kinetic bars both kept at their Section 2 heights — the student's claim of part (d), drawn as a chart — overshoots by exactly its own height, and the tolerance does not cover it: that chart does not earn A3, however short its $\rho g y$ bar. Otherwise both sections are needed, and a total short or over by more than about one unit, 4 mm, at either does not earn it.

EXAMPLE RESPONSE

At Section 2, $5 + 4 + 0 = 9$; at Section 3, $3 + 4 + 2 = 9$. Each chart fills its outline.

MINIMUM SUFFICIENT At both Sections 2 and 3, bars that together reach about the height of the outline. **Not enough:** a total visibly over or short at either section; **also not enough:** Section 3's pressure and kinetic bars both level with Section 2's, beside a $\rho g y$ bar, however short.

Part (b) i *Using the equation of continuity, derive v_2 in terms of v_1, beginning from a fundamental principle or a reference equation.*

For opening with the continuity equation in general symbols, before the areas go in. **Point B1**

Scoring Note. The stem names the equation, so the opening rule applies: $A_1v_1 = A_2v_2$, or Av constant, on the page before the areas are put in. A first line $(2A)\,v_1 = A\,v_2$ has applied it and counts toward B2, but not B1.

EXAMPLE RESPONSE

For the steady flow of a liquid, $A_1v_1 = A_2v_2$.

MINIMUM SUFFICIENT $A_1v_1 = A_2v_2$, or Av constant, in any notation. **Not enough:** $(2A)\,v_1 = A\,v_2$ with nothing above it.

For $v_2 = 2v_1$. **Point B2**

Scoring Note. A bare correct answer earns B2 but not B1. $v_2 = \frac{1}{2}v_1$, the relation turned upside down, is the error the book names beside this part: the narrower section carries the faster flow.

EXAMPLE RESPONSE

$(2A)\,v_1 = A\,v_2$, so $v_2 = 2v_1$.

MINIMUM SUFFICIENT $v_2 = 2v_1$, with no working required. **Not enough:** $v_2 = \frac{1}{2}v_1$.

Part (b) ii *Starting with Bernoulli's equation and using part (b) i, derive P_3 in terms of P_1, ρ, v_1, g and h.*

For opening with Bernoulli's equation in general symbols, before this pipe's speeds go in. **Point B3**

Accept any of these openings:

- $P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_3 + \frac{1}{2}\rho v_3^2 + \rho g y_3$, or the same between Sections 2 and 3
- $P + \frac{1}{2}\rho v^2 + \rho g y$ constant along the flow

Scoring Note. The opening rule of B1 applies. Writing $y_1 = 0$ from the start, so that the first line reads $P_1 + \frac{1}{2}\rho v_1^2 = P_3 + \frac{1}{2}\rho v_3^2 + \rho g h$, is accepted — the series' ruling for terms that drop out (3.2 B1). This point is for the opening only. That $v_3 = v_2 = 2v_1$, because Sections 2 and 3 share an area, is checked at B4, where the answer cannot be reached without it, so forgetting it costs one point, not two.

EXAMPLE RESPONSE

Bernoulli's equation between Sections 1 and 3: $P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_3 + \frac{1}{2}\rho v_3^2 + \rho g y_3$.

MINIMUM SUFFICIENT Bernoulli's equation between two sections in general symbols, zero heights written or left out. **Not enough:** a first line with $2v_1$ already in it; the equation named in words only.

For the correct expression $P_3 = P_1 - \frac{3}{2}\rho v_1^2 - \rho g h$. **Point B4**

Accept any equivalent form:

- $P_1 + \frac{1}{2}\rho v_1^2 - 2\rho v_1^2 - \rho g h$, or $P_1 - \frac{1}{2}\rho\left[(2v_1)^2 - v_1^2\right] - \rho g h$

Scoring Note. Two things go in, and the book names forgetting either as the errors of this part: $v_3 = v_2 = 2v_1$, since Sections 2 and 3 have the same area, and the height term $\rho g h$. A bare correct expression earns B4 alone. Error carried forward from B2 applies: the upside-down $v_2 = \frac{1}{2}v_1$, carried through correctly, gives $P_1 + \frac{3}{8}\rho v_1^2 - \rho g h$ and earns B4. Page 1's exception covers $P_1 - \frac{3}{2}\rho v_1^2$: the height term is what Section 3 is about. $P_1 - \rho g h$ is an error made in this part when it comes from the speed at Section 3 taken as v_1, or from P_2 taken equal to P_1; where the response's own part (b) i gave $v_2 = v_1$, it is what that answer carries, and it earns B4. $P_3 = P_2 - \rho g h$ is true but keeps P_2, which is not among the quantities allowed.

EXAMPLE RESPONSE

With $y_1 = 0$, $y_3 = h$ and $v_3 = v_2 = 2v_1$: $P_1 + \frac{1}{2}\rho v_1^2 = P_3 + \frac{1}{2}\rho\,(2v_1)^2 + \rho g h$, so $P_3 = P_1 + \frac{1}{2}\rho v_1^2 - 2\rho v_1^2 - \rho g h = P_1 - \frac{3}{2}\rho v_1^2 - \rho g h$.

MINIMUM SUFFICIENT $P_3 = P_1 - \frac{3}{2}\rho v_1^2 - \rho g h$ or an equivalent, with no working required — or what the response's own v_2 gives, with the height term present. **Not enough:** $P_1 - \frac{3}{2}\rho v_1^2$; $P_1 - \rho g h$, unless the response's own v_2 is v_1; an answer in terms of P_2.

Part (c) *On axes of speed and of pressure against position along the pipe, from Section 1 through Section 3, with no numerical scale, sketch and label a line for each.*

For the speed: low and level through Section 1, rising where the pipe narrows, then higher and level through Sections 2 and 3, at one height. **Point C1**

Scoring Note. The three points of this part are judged against the pipe, not against the response's earlier parts. The axes carry no scale, so "higher" means visibly higher; the factor of two need not show. The rise may be a step at the boundary or a short ramp. A speed that falls through Section 3 because the pipe climbs is the error the book names beside these graphs: the area there is the same as in Section 2. Each graph has one line, on axes already named, so a missing label costs nothing.

EXAMPLE RESPONSE

A level line at a low value across Section 1, a rise at the narrowing, and a single level line at about twice that height through Sections 2 and 3.

MINIMUM SUFFICIENT Low and level in Section 1, a rise into Section 2, then one level line through Sections 2 and 3. **Not enough:** a speed that falls in Section 3; **also not enough:** a Section 2 speed no higher than Section 1's.

For the pressure from Section 1 to Section 2: highest and level through Section 1, falling where the pipe narrows, then lower and level through Section 2. **Point C2**

Scoring Note. At equal height the faster water has the lower pressure — the drop that pushes it into the narrow part. A pressure that rises into the narrow section, because the water is thought to be squeezed, is the opposite and costs the point. Section 2 is level because neither the speed nor the height changes along it.

EXAMPLE RESPONSE

A level line high on the axis across Section 1, a drop at the narrowing, and a lower level line across Section 2.

MINIMUM SUFFICIENT Higher and level in Section 1, a drop into Section 2, level through Section 2. **Not enough:** a pressure that rises, or stays level, into Section 2.

For the pressure in Section 3: a straight line falling steadily as the water climbs, starting from the Section 2 level. **Point C3**

Scoring Note. The speed no longer changes, but $\rho g y$ grows in proportion to the height, so the pressure falls in proportion to it: $P = P_2 - \rho g y$, a straight line. A level line through Section 3 — the error the book names beside these graphs, and the student's claim in part (d) — costs the point, and so does a line that rises. C3 is marked independently of C2: a correct fall through Section 3 after a wrong Section 2 still earns it.

EXAMPLE RESPONSE

From the Section 2 level, a straight line sloping down across Section 3 to the lowest pressure on the graph.

MINIMUM SUFFICIENT A straight line falling through Section 3 from about the Section 2 level. **Not enough:** a level line through Section 3; **also not enough:** a line that rises, or a curve that flattens out.

Part (d) *A student says that Sections 2 and 3 have the same area, so the water has the same speed in both, and therefore the same pressure. Indicate whether that conclusion about pressure is correct or incorrect, and justify how the answer is consistent with the bar charts of part (a) or the graphs of part (c).*

For a selection that follows from the response's own bar charts or graphs — *Incorrect* where they are right. **Point D1**

Scoring Note. Read this against what the student drew, not against the right answer, as in 3.2 D1 and 6.2 D1. A Section 3 chart whose pressure bar is level with Section 2's — no $\rho g y$ bar, say — makes *Correct* the consistent choice: award D1. That chart has already cost A2 or A3, and one mistake should not be charged three times. This is an exception to page 1's rule on a term left out, made because part (d) asks only that the answer agree with the student's own representation; D2 follows it when the justification reads the bar charts, while a justification read off a pressure graph drawn level through Section 3 earns D1 but not D2 (see D2). Where the bars and the graphs disagree, read the one the justification uses. Where parts (a) and (c) are blank, award D1 for *Incorrect*.

EXAMPLE RESPONSE

Incorrect is marked.

MINIMUM SUFFICIENT One box marked, consistent with the response's own Section 3 bars or pressure graph. **Not enough:** two boxes marked, or none.

For reading the answer off the representation: from Section 2 to Section 3 the kinetic term stays the same and the $\rho g y$ term grows, so with the sum fixed the pressure has to fall — or, on the graphs, the speed line stays level from Section 2 to Section 3 while the water climbs, so the pressure line has to fall. **Point D2**

Scoring Note. The stem asks for consistency with the bar charts or the graphs, so $P_3 = P_2 - \rho g h$ on its own does not earn D2, however correct; it may be added as a check — the ruling of 3.2 D2, 4.2 D2 and 6.2 D2. Reading the quantities the bars stand for counts as reading the bars: "the kinetic term is the same, but $\rho g y$ is larger at Section 3" is enough without the word "bar". Both halves are needed, what stays and what changes, as the book's tip beside this part says. Error carried forward: read the argument against the response's own bar chart; a student whose Section 3 chart has no $\rho g y$ bar, and who argues from it that equal kinetic bars and an unchanged total leave the pressure the same, earns D2. A pressure graph shows no height term, so on the graphs the climb has to be in the argument: a pressure line drawn level through Section 3 and read back as "same speed, so same pressure" is the claim under test, not a reading, and does not earn D2.

EXAMPLE RESPONSE

On my charts the $\frac{1}{2}\rho v^2$ bars at Sections 2 and 3 are the same height — same area, same speed — but Section 3 has a $\rho g y$ bar that Section 2 does not. The three bars add up to the same total, so Section 3's pressure bar is shorter: $P_3 < P_2$, and the student is wrong. (Check: $P_3 = P_2 - \rho g h$.)

MINIMUM SUFFICIENT One sentence: the same kinetic term at Sections 2 and 3, a larger $\rho g y$ at Section 3, so a lower pressure there — or, from the graphs: the speed level through Sections 2 and 3, the water higher in Section 3, so the pressure line falls there — or, read against the response's own chart at Section 3: with no $\rho g y$ bar there, equal kinetic bars and an equal total leave the pressure bars equal. **Not enough:** $P_3 = P_2 - \rho g h$ with no words; **also not enough:** "the water is higher up" with nothing on how that changes the pressure, or "my pressure graph goes down" with nothing on the climb.

Set B — Reinforcement

Problem 8.5 • 8 rubric points • a second Qualitative Quantitative Translation problem, about eighteen minutes: two tanks of different liquids drain through the same hole from the same height — a supplement to Set A, not a paper of its own. Its tally sheet is the last page.

Problem 8.5: Qualitative Quantitative Translation (QQT) — 8 rubric points

The problem. Two large open tanks are filled to the same height H: Tank 1 with a liquid of density ρ_1, Tank 2 with one of density $\rho_2 > \rho_1$. Each has a small hole of the same area in its bottom, much smaller than the tank's cross-section; v_1 and v_2 are the speeds at which the liquids leave the holes when they are opened.

Part (a) *Indicate whether v_1 is greater than, less than or equal to v_2, and justify in terms of the energy of a small volume of liquid moving from the surface to the hole, with qualitative reasoning beyond referencing equations.*

For selecting $v_1 = v_2$. — Point A1

Scoring Note. The selection alone earns this point.

EXAMPLE RESPONSE

$v_1 = v_2$ is marked.

MINIMUM SUFFICIENT The $v_1 = v_2$ box marked, ticked or circled, or the choice written out unambiguously in words. **Not enough:** more than one box marked.

For what is the same in both tanks: the small volume falls through the same height, H, and it starts and ends at the same pressure, the air's, at the open surface and at the hole. — Point A2

Scoring Note. The book's answer names both halves, and both are needed: the drop, and the air pressure at the two ends — with the same pressure where the volume starts and where it leaves, the energy it gains comes from the drop alone. For the second half the hole has to be named along with the surface: "open to the air at the top and at the hole" is enough, while "both tanks are open" says nothing about where the liquid leaves. A larger pressure near the bottom of the denser liquid is no reason for a faster jet; that is the error the book names beside this part, and A3 is where it is answered.

EXAMPLE RESPONSE

In both tanks the small volume of liquid falls through the same height H, and it starts at the open surface and leaves at the hole, both at atmospheric pressure.

MINIMUM SUFFICIENT The same height H, and atmospheric pressure at both the surface and the hole — both halves. **Not enough:** "the tanks are filled to the same height" alone; **also not enough:** "both tanks are open", with nothing on the hole.

For why the density makes no difference: the denser liquid's volume has more mass, so it gives up more gravitational energy in the drop, but it needs proportionally more kinetic energy to reach any given speed — both grow with the density, so the speed is the same. **Point A3**

Scoring Note. What changes is the density, and the response has to say why that does not change the speed: the energy released and the energy needed both go up in proportion to the mass of the volume. Only one half — "more mass, so more energy" — argues for a faster jet, and so does the larger pressure at the bottom of the denser liquid, the error the book names here: it looks only at what changes. The book also states the balance in terms of pressure: in the denser liquid the pressure difference over the depth is larger, so each volume is handed proportionally more energy — the pressure term of Bernoulli's equation is energy per volume — but it has proportionally more mass and needs proportionally more kinetic energy for the same speed; put that way, in energy, it earns A3 too. A larger push on a larger mass — the pressure's or gravity's — with no energy in it answers in forces, not in the energy the stem asks for, and does not. A comparison with free fall earns A3 when it is put in energy terms, both energies in proportion to the mass; "everything falls at the same rate" alone does not, since the stem asks for energy. "The density cancels", quoted from the equation, is the conclusion of part (c), not reasoning, and does not earn it either.

EXAMPLE RESPONSE

The denser liquid packs more mass into the same small volume, so as the volume falls through H it gives up more gravitational energy — but that larger mass also needs proportionally more kinetic energy to move at any given speed. The energy released and the energy required grow in the same proportion with the density, so the two liquids leave at the same speed.

MINIMUM SUFFICIENT One sentence: more mass means more gravitational energy released but also more kinetic energy needed, in the same proportion, so the speed does not change — or the same balance in pressure terms: more pressure energy per volume from the larger pressure difference over H, matched by proportionally more kinetic energy needed. **Not enough:** "the denser liquid has more pressure at the bottom, so it is faster"; a larger push on a larger mass, with no energy in it; **also not enough:** "the density cancels" with no reason.

Part (b) *Starting with Bernoulli's equation, derive the speed v at which a liquid of density ρ leaves a small hole in the bottom of a large open tank filled to height H, in terms of H and g.*

For opening with Bernoulli's equation in general symbols, between the surface and the hole. **Point B1**

Accept any of these openings:

- $P_s + \frac{1}{2}\rho v_s^2 + \rho g y_s = P_h + \frac{1}{2}\rho v^2 + \rho g y_h$
- $P + \frac{1}{2}\rho v^2 + \rho g y$ constant along the flow

Scoring Note. The stem names the equation, so the opening rule applies. A first line $\rho g H = \frac{1}{2}\rho v^2$ has applied it, with every condition already used, and counts toward B3 but not B1. Energy conservation for a falling volume, $mgH = \frac{1}{2}mv^2$, is sound physics by another road: it earns what a bare answer earns, B3, never B1.

EXAMPLE RESPONSE

Bernoulli's equation from the surface (s) to the hole (h): $P_s + \frac{1}{2}\rho v_s^2 + \rho g y_s = P_h + \frac{1}{2}\rho v^2 + \rho g y_h$.

MINIMUM SUFFICIENT Bernoulli's equation between the surface and the hole in general symbols. **Not enough:** $\rho g H = \frac{1}{2}\rho v^2$ with nothing above it.

For the pressure terms: the surface and the hole are both open to the air, so $P_s = P_h = P_{\text{atm}}$ and the two cancel. **Point B2**

Scoring Note. Losing track of the fact that the surface and the hole are at the same pressure is the error the book names beside this part. The version of it to look for puts the pressure of the still liquid, $P_{\text{atm}} + \rho g H$, at the hole: it cancels the very term that drives the flow and leaves $v = 0$. The book also neglects the surface's speed, because the tank is much wider than the hole; that step is judged at B3, where the answer depends on it.

EXAMPLE RESPONSE

The surface and the hole are both at atmospheric pressure: $P_s = P_h = P_{\text{atm}}$, and these terms cancel.

MINIMUM SUFFICIENT $P_s = P_h$, both atmospheric — written, or shown by the pressure terms dropping out of the line after the opening. **Not enough:** $P_{\text{atm}} + \rho g H$ at the hole.

For $v = \sqrt{2gH}$. **Point B3**

Scoring Note. Two more conditions go in here: the surface's speed is negligible, because the tank is much wider than the hole, and the surface is H above the hole; then ρ cancels. A bare correct expression earns B3 alone, and so does energy conservation for a falling volume. Keeping ρ in the answer — the other error the book names here — does not earn B3, nor does $v^2 = 2gH$ left unsolved. The $v = 0$ that follows from the still liquid's pressure at the hole is fixed by that wrong condition and is not carried.

EXAMPLE RESPONSE

With $v_s \approx 0$, $y_s = H$ and $y_h = 0$: $\rho g H = \frac{1}{2}\rho v^2$, so $v^2 = 2gH$ and $v = \sqrt{2gH}$.

MINIMUM SUFFICIENT $v = \sqrt{2gH}$, with no working required. **Not enough:** an expression that still contains ρ; $v^2 = 2gH$.

Part (c) *Indicate whether the expression from part (b) is consistent with the claim in part (a), and briefly justify by referring to specific terms of the derivation.*

For selecting *Consistent*. **Point C1**

Scoring Note. The ruling of 8.2 C1: the selection alone, and *Not consistent* only where the response says outright that its own expression disagrees with its part (a) claim.

EXAMPLE RESPONSE

Consistent is marked.

MINIMUM SUFFICIENT The *Consistent* box marked, ticked or circled, or the choice written out unambiguously in words — or *Not consistent* where the response says outright that its own expression contradicts its part (a) claim. **Not enough:** both boxes marked, or no choice at all.

For using what the expression leaves out: $\sqrt{2gH}$ has no ρ in it — the density cancelled — so with the same H and g the two tanks give the same speed, $v_1 = v_2$. **Point C2**

Accept any of these:

- v depends only on g and H, which the tanks share
- ρ appears on both sides of Bernoulli's equation and cancels
- on a carried-forward expression that keeps ρ, a correct reading of how its v changes with ρ, whatever that says about part (a)

Scoring Note. The book's tip beside this part makes the point: here the deciding feature is a term that is missing. "Both tanks have the same H" alone does not settle it — the tanks differ in density, and the expression has to be shown not to care. Error carried forward, as in 8.2 C2: read the response's own expression.

EXAMPLE RESPONSE

$v = \sqrt{2gH}$ contains g and H but no density: ρ cancelled in the derivation. Both tanks have the same H, so the expression gives them the same exit speed, $v_1 = v_2$, the claim of part (a).

MINIMUM SUFFICIENT A sentence saying that ρ is not in the expression, or that v depends only on g and H, and so $v_1 = v_2$ — or, on a carried-forward expression, a sentence reading how its v depends on ρ. **Not enough:** "yes, consistent"; "same H, so the same speed", with nothing on the density.

Tally sheet • Set A

Problems 8.1–8.4, forty points — a complete free-response paper. One box per point, in order. Photocopy it, or keep one on the desk and write the student's name at the top. The short labels are reminders, not the rubric — the ruling for each point is on its own page above.

8.1 MR 10 pts

- ☐ **A1** F_B up, Mg down, T down, labelled, no extras (face arrows in place of F_B accepted)
- ☐ **A2** F_B the longest, about $T + Mg$ (within a seventh); T or Mg longer, either way
- ☐ **A3** opens with $P = P_0 + \rho gh$ in general symbols
- ☐ **A4** $F_B = \rho_L g s^3$ — no d left, area included
- ☐ **A5** opens with $\sum F = ma$, $a = 0$ (or $\sum F = 0$)
- ☐ **A6** $T = \rho_L g s^3 - Mg$, or ecf — not $Mg - \rho_L g s^3$, not $\rho_L g s^3$, not 0
- ☐ **A7** 7.0 N (6.9 with $g = 9.8$) with unit, or ecf — not 12 (11.8), not 0
- ☐ **B1** "Less than"
- ☐ **B2** F_B unchanged, with a reason: same volume of the same liquid
- ☐ **B3** fixed F_B = weight + tension, so a heavier cube means less tension, in words

8.2 QQT 8 pts

- ☐ **A1** "Less than"
- ☐ **A2** faster where the pipe is narrower — the narrowing named
- ☐ **A3** speeds up ⇒ net force downstream ⇒ push from behind larger, $P_1 > P_2$ — not Bernoulli quoted
- ☐ **B1** opens with Bernoulli in general symbols
- ☐ **B2** $v_2 = (A_1/A_2)\, v_1$ — not upside down
- ☐ **B3** $\frac{1}{2}\rho v_1^2[(A_1/A_2)^2 - 1]$, or ecf — no v_2 left
- ☐ **C1** "Consistent" (or a contradiction it states)
- ☐ **C2** $(A_1/A_2)^2 - 1 > 0$ as $A_1 > A_2$, so $P_1 > P_2$ — or reads *their own* expression

8.3 LAB 10 pts

- ☐ **A1** gauge pressure at several depths, each measured with the meter-stick
- ☐ **A2** a real uncertainty-reducing step (repeat and average; depth to the tip), not just several depths
- ☐ **B1** a linear pair: P_g and h, either way round, or another
- ☐ **B2** ρ = slope/g, or the conversion for its axes
- ☐ **C1** P_g and h in the blanks (or another accepted pair)
- ☐ **C2** both axes: name and unit; uniform scale where renumbered
- ☐ **C3** at least four of the five points in place
- ☐ **C4** one straight line through the points
- ☐ **D1** slope from two points *on the line*
- ☐ **D2** ρ from the slope via its axes; 1.05–1.35 × 10^3 kg/m^3 if D1 earned

8.4 TBR 12 pts

- ☐ **A1** Sec. 2: kinetic bar 3–5 times Sec. 1's; no ρgy bar
- ☐ **A2** Sec. 3: kinetic bar level with *their own* Sec. 2 bar; a ρgy bar
- ☐ **A3** Sec. 2 and Sec. 3 each add up to the outline, 9 units — not Sec. 2's P and kinetic bars kept beside a ρgy bar
- ☐ **B1** opens with $A_1 v_1 = A_2 v_2$
- ☐ **B2** $v_2 = 2v_1$
- ☐ **B3** opens with Bernoulli in general symbols
- ☐ **B4** $P_3 = P_1 - \frac{3}{2}\rho v_1^2 - \rho gh$, or ecf — ρgh present
- ☐ **C1** v: low in Sec. 1, a rise, one higher level through Sec. 2 and 3
- ☐ **C2** P: high and level in Sec. 1, a drop, lower and level in Sec. 2
- ☐ **C3** P in Sec. 3: a straight line falling from the Sec. 2 level
- ☐ **D1** "Incorrect" (or what *their own* Sec. 3 chart or graph gives)
- ☐ **D2** same kinetic term (level speed), larger ρgy (the climb) ⇒ lower P — bars or graphs; ecf on *their own* bars

SET A TOTAL

8.1 _____ / 10
8.2 _____ / 8
8.3 _____ / 10
8.4 _____ / 12

_____ / 40

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Tally sheet • Set B

Problem 8.5, eight points — the chapter's second Qualitative Quantitative Translation problem. Same arrangement as Set A: one box per point, in order, with short reminders rather than the rulings themselves.

8.5 QQT 8 pts

- ☐ **A1** "$v_1 = v_2$"
- ☐ **A2** same drop H *and* atmospheric pressure at the surface and at the hole (not just "open tanks")
- ☐ **A3** more mass: more energy released (by the drop, or as pressure) *and* more needed, in proportion — so the same speed; in energy, not forces
- ☐ **B1** opens with Bernoulli in general symbols, surface to hole
- ☐ **B2** $P_s = P_h$, both atmospheric — not $P_{\text{atm}} + \rho g H$ at the hole
- ☐ **B3** $v = \sqrt{2gH}$ — no ρ, not v^2
- ☐ **C1** "Consistent" (or a contradiction it states)
- ☐ **C2** no ρ in $\sqrt{2gH}$, the same H, so $v_1 = v_2$ — or reads *their own* expression

SET B TOTAL

8.5 _____ / 8

_____ / 8

The two sheets together come to forty-eight points, the chapter total. Only Set A is a complete paper; Set B adds one problem for a class ready to see why a tank drains at the same speed whatever liquid is in it.

Scoring key, Unit 8: Fluids — Set B. Written for *AP Physics 1: Algebra-Based — Practice Workbook*, Chapter 8, problem 8.5. Original rubrics for original problems, set out in the format of published scoring guidelines. Not a College Board document and not endorsed by College Board. AP® is a registered trademark of College Board, which was not involved in producing this material.